Last name ______________________

First name ______________________

LARSON—MATH 556–HOMEWORK WORKSHEET 10
Dilworth and Line Coloring

More Dilworth

1. Let G be bipartite. Kőnig's Minimax Theorem tells us that $\tau = \nu$. Show this is equivalent to $\alpha + \nu = |V(G)|$.

2. (a) Draw the *milkbone* graph. Call it $G = (A, B)$. It's bipartite, so define appropriate sets A, B.
   (b) Let $P = (V(G), \preceq)$ be the poset where the relation "$\preceq$" is defined as:
   $$x \preceq y \Leftrightarrow x \in A, y \in B \text{ and } xy \in E(G).$$
   Draw the Hasse diagram for P.
   (c) Find a largest antichain $\mathcal{A}$ in P.
   (d) Check that $\mathcal{A}$ is an independent set in G.
   (e) Argue that there can't be a larger independent set in G (so $\alpha(G) = |\mathcal{A}|$).
   (f) Find a minimum chain decomposition $\mathcal{C}$ in P.
   (g) The chains in $\mathcal{C}$ have at most two elements. Let $\mathcal{C}'$ be the collection of two-element chains.
   (h) Explain why $\mathcal{C}'$ corresponds to a matching in G.
   (i) Argue that there can't be a larger matching in G (so, $\nu(G) = |\mathcal{C}'|$).
   (j) Let $\mathcal{B} = \{y \in V(G) : \{x, y\} \in \mathcal{C}' \text{ and } x \in \mathcal{A}\}$.
   (k) Explain why $|\mathcal{C}'| = |\mathcal{B}|$ (so $\nu(G) = |\mathcal{B}|$).
   (l) Check that $\mathcal{A}, \mathcal{B}$ are a partition of $V(G)$.
   (m) Argue (explain) why $\alpha(G) + \nu(G) = |V(G)|$.

Kőnig's Line Coloring Theorem

3. Find a tree that has more than one (different) maximum matching.

4. **Show:** If a tree has a perfect matching then it is unique.

5. We showed that regular bipartite graphs have perfect matchings. Is it true in general that regular graphs have perfect matchings? What can you say here?

6. What is the line coloring number (*chromatic index*) χ_e of the Petersen graph? (And, of course, argue that your claim is correct!)