Last name ______________________

First name ______________________

LARSON—MATH 556–HOMEWORK WORKSHEET 08
Spanning Trees

A **spanning tree** T of a connected graph G is a subgraph that is a tree and contains all the points of G (so $V(T) = V(G)$ and $E(T) \subseteq E(G)$).

For instance, let C_n be a cycle on n points and T be the graph formed by removing any singe edge of C_n; then T is a spanning tree of C_n.

We **proved**: Every connected graph has a spanning tree. That is, prove if G is a connected graph then there is a subgraph T of G which is a tree and includes all of the points of T.

1. Find a spanning tree for each of the following graphs:
   (a) K_5.
   (b) $K_{3,4}$.
   (c) The house graph.
   (d) The bow-tie graph.
   (e) The Petersen graph
2. Let G be the Petersen graph. Find a spanning tree T with $\nu(G) = \nu(T)$.
3. Let G be a connected graph. Let M be a maximum matching of G. So M is a forest. Let F be a maximum forest containing all the lines of M (that is, there is no forest F' with all the lines of M and more lines than F). Prove that F is connected.
4. Prove: every connected graph G has a spanning tree T with $\nu(G) = \nu(T)$.

Kőnig & Dilworth

5. Prove: Dilworth's Theorem implies Kőnig's Minimax Theorem.