Last name ______________________

First name ______________________

LARSON—MATH 556–HOMEWORK WORKSHEET 07
Test 1 Review

You should know the following definitions, theorems, algorithms, and proofs for the test. Write out careful definitions, theorem statements, algorithms, proofs, and solutions. Turn these in at test time.

Definitions. Write each definition and give an example.

1. A **matching** in a graph.
2. A **perfect matching**.
3. The **matching number**.
4. A **point cover**.
5. The **point covering number**.
6. A **line cover**.
7. The **line covering number**.
8. An **independent set** of points.
9. The **independence number**.
10. A **clique**.
11. The **clique number**.
12. A **bipartite** graph.
13. A **complete** graph.
14. A **complete bipartite** graph.
15. A **path**.
16. A **cycle**.
17. A **Hamilton** cycle.
18. A **connected** graph.
19. An **induced subgraph**.
20. The **symmetric difference** of sets A and B.
21. A **graph property**.
22. An **NP-property**.

23. A **minimax theorem**.

24. An M-**alternating path**.

25. An M-**augmenting path**.

26. **adjacency matrix** of a graph.

27. The **complement** of a graph.

28. A **tree**.

29. A **spanning tree** (of a connected graph).

30. **system of distinct representatives**.

Theorems. State and give an example, application or confirmation.

31. The **Gallai Identities**.

32. **König's Minimax Theorem**.

33. **Hall's Theorem**.

34. The **Frobenius Marriage Theorem**.

35. **Berge's** Theorem.

36. **Hall's** SDR Theorem.

Notation

37. Define ν, τ, α, ρ, ω, $V(G)$, $E(G)$, $K_{m,n}$, K_n, $G[X]$ for $X \subset V(G)$, $A \oplus B$, C_n, S_n, $\Gamma(X)$ for $X \subset V(G)$

Graphs

38. Draw the Petersen graph, the house, the milkbone, $K_{3,4}$, K_5, C_5, S_5, and P_5.

Algorithms

39. Carefully state an algorithm for finding a maximum independent set in a graph. Explain why it works.

40. What is the input and output of one iteration of the Hungarian Method?

Proofs

41. Prove: $\alpha(K_n) = 1$.

42. Prove: $\tau(K_{m,n}) = min\{m, n\}$.

43. Prove: A subgraph of a bipartite graph is bipartite.

44. Prove: For any graph G $\alpha(G) + \tau(G) = |V(G)|$.

45. Prove: For any graph G, $\nu(G) \leq \tau(G)$.

46. Prove: Berge's Theorem.

47. Prove: every tree is bipartite.

Problems. Explain your answers.

48. Find an adjacency matrix of the house graph.

49. u_1 can do jobs v_2 and v_3. u_2 can do jobs v_1, v_2, v_3 and v_4. u_3 can do jobs v_2 and v_3. u_4 can do jobs v_2 and v_3. Is there a possible assignment of the employees to the jobs so that all the jobs get done? Find an assignment, or argue that no assignment is possible.

50. Give an example of a graph and a matching which is not maximal. Explain why it is not.

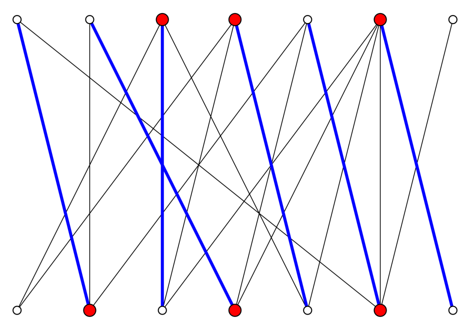
51. Give an example of a graph and a matching which is maximal but not maximum.

52. Give an example of a graph which does not have a Hamilton cycle. Explain why it does not have one.

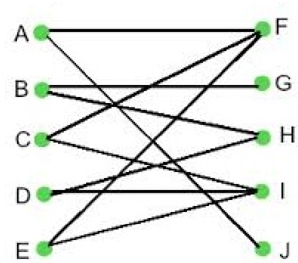
53. Give an example of a graph G and a subgraph H of G which is **not** induced. Explain why it is not.

54. Calculate α for each of the graphs in the GRAPHS section above. Argue that your answer is correct.

55. What is the **importance** of minimax theorems?



56. Find a maximum matching and a minimum point cover in the above graph. **Argue** that your matching is maximum.



57. Find a maximum matching and a minimum point cover in the above graph. **Argue** that your matching is maximum.