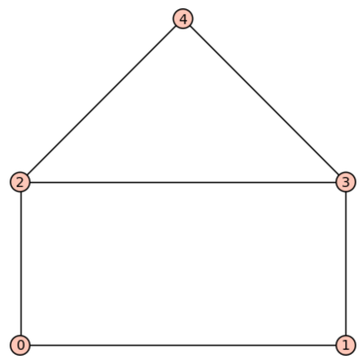
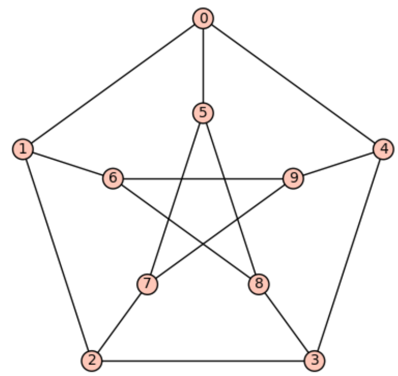
Last name ______________________

First name ______________________

LARSON—MATH 556–HOMEWORK WORKSHEET 05
Complements and Cliques





The **complement** of a graph G is a graph $\bar{G}$ with the same set of points (so $V(\bar{G}) = V(G)$ and with lines $E(\bar{G}) = \{vw : v, w \in V(\bar{G})$ and $vw \notin E(G)\}$ (that is, vw is a line in $\bar{G}$ if and only if it is not a line in G).

1. Draw P_3 and $\bar{P}_3$.
2. Draw P_5 and $\bar{P}_5$.
3. Draw C_5 and $\bar{C}_5$.
4. Draw the complement of the house graph.
5. Draw the complement of the Petersen graph.
6. The **size** of a graph is the number of lines of the graph. State and prove a relationship between the size of a graph G and the size of its complement $\bar{G}$.

   A **clique** in a graph is a complete subgraph (so the points $S \subseteq V(G)$ induce a clique in graph G if and only if G has a line between every pair of points of S). A clique in a graph is maximal if it is not contained in a larger clique. A clique is maximum if it has more points than any other clique.

7. Find all 4 maximal cliques in the house graph.
8. Find the (unique) maximum clique in the house graph.

   The **clique number** ω of a graph is the cardinality of a maximum clique.

9. Find ω for the house graph.
10. Find ω for the Petersen graph.
11. Find formulas for $\omega(P_n)$, $\omega(C_n)$, $\omega(S_n)$ and $\omega(K_n)$.
12. Prove: For any graph G, $\omega(G) = \alpha(\bar{G})$.
13. Prove: For any graph G, $\omega(G) + \alpha(G) \leq |V(G)| + 1$.