Last name ______________________

First name ______________________

LARSON—MATH 556–HOMEWORK WORKSHEET 04
HALL—FROBENIUS EQUIVALENCE

For proofs, write out the definitions as needed, explain your notation, and be extremely clear. The goal of a proof is to convince **other** readers of your argument. Write as if you are writing to your colleagues. Remember that no one reads minds—they only can know what you tell them.

Hall's Theorem Let $G = (A, B)$ be a bipartite graph. Then G has a matching of A into B (a matching that covers the points in A) if and only if $|\Gamma(X)| \geq |X|$ for all $X \subseteq A$.

Frobenius' (Marriage) Theorem A bipartite graph $G = (A, B)$ has a perfect matching if and only if $|A| = |B|$ and $|\Gamma(X)| \geq |X|$ for each $X \subseteq A$.

1. Prove: Hall's Theorem implies Frobenius' Theorem.

   Assume Hall's Theorem. Suppose $G = (A, B)$ is a bipartite graph with partite sets (color classes) A and B.

   (a) Assume G has a perfect matching. Show: $|A| = |B|$ and $|\Gamma(X)| \geq |X|$ for each $X \subseteq A$.

   (b) Assume $|A| = |B|$ and $|\Gamma(X)| \geq |X|$ for each $X \subseteq A$. Show: G has a perfect matching.

2. Prove: Frobenius' Theorem implies Halls' Theorem.

   Assume Frobenius' Theorem. Suppose $G = (A, B)$ is a bipartite graph with partite sets (color classes) A and B.

   (a) Assume G has a matching of A into B. Show: $|\Gamma(X)| \geq |X|$ for each $X \subseteq A$.

   (b) Assume $|\Gamma(X)| \geq |X|$ for each $X \subseteq A$. Show: G has a matching of A into B.