Last name _____

First name _____

LARSON—MATH 556–HOMEWORK WORKSHEET 03

For proofs, write out the definitions as needed, explain your notation, and be extremely clear. The goal of a proof is to convince **other** readers of your argument. Write as if you are writing to your colleagues. Remember that no one reads minds—they only can know what you tell them.

1. Prove: If G is a bipartite graph then any subgraph of G is bipartite.
2. State an algorithm for finding a maximum matching in a graph. Prove that it works (that is, that it will always yield a maximum matching).
3. Use König's Theorem and a Gallai Identity to prove: if G is bipartite then $\alpha(G) = \rho(G)$.
4. Show that the graph G in the following figure is bipartite. Let the partite sets be A and B. Check that $|A| = |B|$.

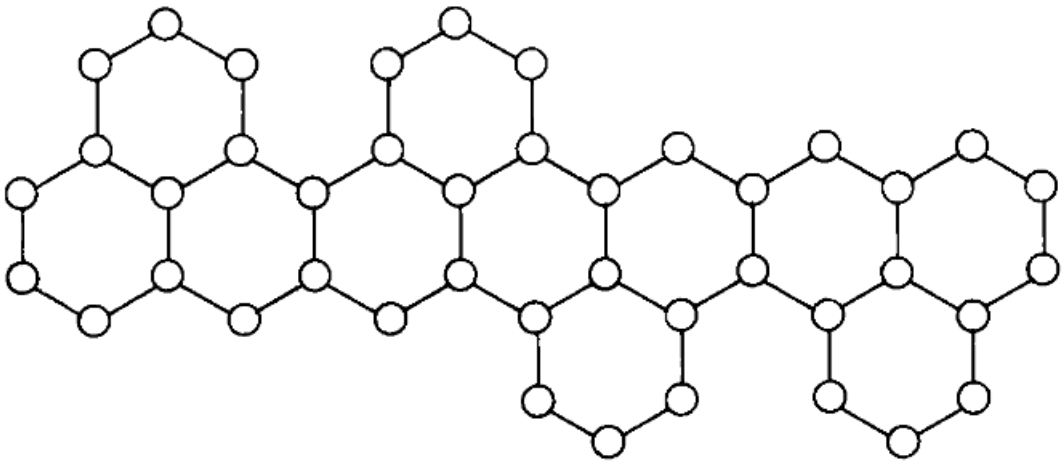

FIGURE 1.1.1.

5. (LP Exercise 1.1.6) Find a perfect matching in the previous graph or find a set $X \subseteq A$ with $|\Gamma(X)| < |X|$.