Last name _____

First name _____

LARSON—MATH 556–HOMEWORK WORKSHEET 01
Four Fundamental Invariants

For **proofs**, write out the definitions as needed, explain your notation, and be extremely clear. The goal of a proof is to convince **other** readers of your argument. Write as if you are writing to your colleagues. Remember that no one reads minds—they only can know what you tell them.

The **complete bipartite graph** $K_{n,m}$ consists of two sets of points A and B, with $|A| = n$ and $B| = m$, where every point in A is adjacent to every point in B.

The **cycle** C_n consists of n points $V(C_n) = \{v_1, \ldots, v_n\}$, and n lines $E(C_n) = \{v_1v_2, \ldots, v_{n-1}v_n, v_nv_1\}$.

The **complete graph** K_n consists of n points where every pair of points is adjacent.

1. **Matching Number**

   (a) Draw the complete bipartite graph $K_{3,4}$, find ν, and then *argue* that your answer is correct.

   (b) Conjecture a formula for $\nu(K_{n,m})$.

   (c) Draw the cycle C_4 , find ν, and then *argue* that your answer is correct.

   (d) Draw the cycle C_5 , find ν, and then *argue* that your answer is correct.

   (e) Conjecture a formula for $\nu(C_n)$.

   (f) Draw the complete graph K_4, find ν, and then *argue* that your answer is correct.

   (g) Draw the complete graph K_5, find ν, and then *argue* that your answer is correct.

   (h) Conjecture a formula for $\nu(K_n)$.

2. **Line Covering Number**

   (a) Find $\rho(K_{3,4})$, and then *argue* that your answer is correct.

   (b) Conjecture a formula for $\rho(K_{n,m})$.

   (c) Find $\rho(C_4)$, and then *argue* that your answer is correct.

   (d) Find $\rho(C_5)$, and then *argue* that your answer is correct.

   (e) Conjecture a formula for $\rho(C_n)$.

   (f) Find $\rho(K_4)$, and then *argue* that your answer is correct.

   (g) Find $\rho(K_5)$, and then *argue* that your answer is correct.

   (h) Conjecture a formula for $\rho(K_n)$.

3. **Independence Number**

   (a) Find $\alpha(K_{3,4})$, and then *argue* that your answer is correct.
   (b) Conjecture a formula for $\alpha(K_{n,m})$.
   (c) Find $\alpha(C_4)$, and then *argue* that your answer is correct.
   (d) Find $\alpha(C_5)$, and then *argue* that your answer is correct.
   (e) Conjecture a formula for $\alpha(C_n)$.
   (f) Find $\alpha(K_4)$, and then *argue* that your answer is correct.
   (g) Find $\alpha(K_5)$, and then *argue* that your answer is correct.
   (h) Conjecture a formula for $\alpha(K_n)$.

4. **Point Covering Number**

   (a) Find $\tau(K_{3,4})$, and then *argue* that your answer is correct.
   (b) Conjecture a formula for $\tau(K_{n,m})$.
   (c) Find $\tau(C_4)$, and then *argue* that your answer is correct.
   (d) Find $\tau(C_5)$, and then *argue* that your answer is correct.
   (e) Conjecture a formula for $\tau(C_n)$.
   (f) Find $\tau(K_4)$, and then *argue* that your answer is correct.
   (g) Find $\tau(K_5)$, and then *argue* that your answer is correct.
   (h) Conjecture a formula for $\tau(K_n)$.

5. **Prove** one of your conjectures.