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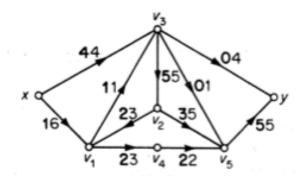
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## LARSON—MATH 556—CLASSROOM WORKSHEET 26 Max Flow-Min Cut Theorem

## Review

- What is a *network*?
- What is a *flow* in a network?
- What is the *value* of a flow in a network?
- What is the *capacity* of a cut  $\nabla^+(A)$  (or a separator A) in a network?
- What is an *f*-augmenting path to  $u_k$  in a network? What is an *f*-augmenting path in a network?

## **Network Flows**



1. Does this network have a flow-augmenting path?

2. (Claim:) A flow f is maximum if and only if there are no f-augmenting paths.

3. (Max-Flow Min-Cut Theorem:) The value of a maximum flow in a network equals the capacity of a minimum cut.

4. (Flow Integrality Theorem:). If the capacities of a network are integers, then there exists a maximum flow which is integral on every line.

## A Max Flow-Min Cut Proof of Kőnig's Minimax Theorem

5. Let G be the milkbone graph. It bipartite, so G = (A, B). Build a network G' by directing all lines of G from A to B, adding a new point s (the "source") joined to all points of A and a new point t (the "sink") to which all points of B are joined, and then assigning capacity  $\infty$  to all lines of G and capacity 1 to all new lines of G'.

6. Find a maximum flow. What do you notice?

7. Can you see how to generalize this example to give a new proof of Kőnig's Theorem?