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LARSON—MATH 556—CLASSROOM WORKSHEET 18 Partially Ordered Sets & Dilworth's Theorem

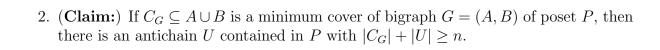
Review

- A partial order on a set X is a relation " \leq " on X that is reflexive, anti-symmetric and transitive. We call (X, \leq) a partially ordered set (or poset).
- For $x, y, z \in X$, x covers y (or equivalently y is covered by x) if $y \le x$ and $y \le z \le x$ implies that z = x or z = y. A Hasse diagram (or covering diagram) for a poset (X, \le) is a representation of the elements of X together with a line between elements x and y is x covers y.
- A chain in (X, \leq) is a linearly ordered subset of X (with respect to the given order \leq), that is $C \subseteq X$ and (C, \leq) is a linear order.
- An anti-chain in a poset is a collection of elements which are pair-wise incomparable.
- (Divisibility poset). Let $X = \{2, 3, ... 10\}$ and define the divisibility relation "|": For $x, y \in X$, x|y (that is, x divides y, or y is divisible by x)) if there is an integer k such that kx = y. Show that (X, |) is a poset.
- We showed "|" is reflexive, anti-symmetric and transitive.
- What is the bipartite graph G Lovasz and Plummer associate with a poset P?

Proof of Dilworth's Theorem

Let P be a poset with elements $X = \{x_1, \ldots, x_n\}$ and corresponding bipartite graph G = (A, B).

1. (**Lemma:**) Let M be a matching in G. Then there is a chain partition \mathcal{P} of P such that $|M| + |\mathcal{P}| = n$.



3. (**Dilworth's Theorem:**) In any finite partially ordered set the cardinality of any largest antichain equals the cardinality of the smallest chain partition.

5. What is the Birkhoff non-Neumann Th	veorem?	

4. What is a double stochastic matrix?

6. What is Birkhoff's Algorithm?