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LARSON—MATH 556—CLASSROOM WORKSHEET 16 Partially Ordered Sets & Dilworth's Theorem

Homework

• A spanning tree in a connected graph is a subgraph that is a tree that contains all the points of the parent graph. Prove that every connected graph has a spanning tree.

Review

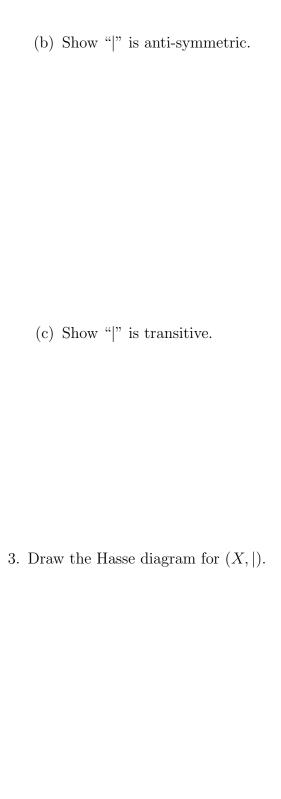
- A partial order on a set X is a relation " \leq " on X that is reflexive, anti-symmetric and transitive. We call (X, \leq) a partially ordered set (or poset).
- 1. For $x, y, z \in X$, x covers y (or equivalently y is covered by x) if $y \le x$ and $y \le z \le x$ implies that z = x or z = y. A Hasse diagram (or covering diagram) for a poset (X, \le) is a representation of the elements of X together with a line between elements x and y is x covers y.

A chain in (X, \leq) is a linearly ordered subset of X (with respect to the given order \leq), that is $C \subseteq X$ and (C, \leq) is a linear order.

An anti-chain in a poset is a collection of elements which are pair-wise incomparable.

Dilworth's Theorem says that the number of elements in a largest anti-chain in a poset equals the number of chains in a smallest chain decomposition.

- 2. (**Divisibility poset**). Let $X = \{2, 3, ... 10\}$ and define the divisibility relation "|": For $x, y \in X$, x|y (that is, x divides y, or y is divisible by x)) if there is an integer k such that kx = y. Show that (X, |) is a poset.
 - (a) Show "|" is reflexive.



4. Find examples of chains in $(X,)$.
5. Find a partition of the elements of $(X,)$ into a minimum number of chains.
6. Find a largest anti-chain in $(X,)$.
Dilworth's Theorem
7. What is the bipartite graph G Lovasz and Plummer associate with a poset P ?

8. Draw this graph for our divisibility poset example.

Let P be a poset with elements $X = \{x_1, \dots, x_n\}$ and corresponding bipartite graph G = (A, B).

9. (Claim:) Let M be a matching in G. Then there is a chain partition \mathcal{P} of P such that $|M| + |\mathcal{P}| = n$.