Last name _____

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LARSON—MATH 556—CLASSROOM WORKSHEET 09
NP-properties and the Hungarian Method

Concepts & Notation

- *assignment problem*, *graph* G, *points* $V(G)$, *lines* $E(G)$, *adjacent*, *incident*.
- *line covering*, *line covering number* ρ, *matching*, *matching number* ν, point covering, *point covering number* τ, *independent set*, *independence number* α.

Review

- **König's Theorem:** For any bipartite graph, $\tau = \nu$.
- A **perfect matching** (or **1-factor**) is a matching which covers all points of G.
- What is *Hall's Theorem*?
- What is *Frobenius's (Marriage) Theorem*?
- What is a *Hamilton cycle* in a graph?
- What's an algorithm for finding a Hamilton cycle in a graph?
- What is the conceptual difference between the problems (1) of finding a perfect matching in a bipartite graph, and (2) finding a Hamilton cycle in a graph?
- Why is "having a perfect matching" an *NP-property* of a graph?
- Why is "having a set of points X where $|X| > |\Gamma(X)|$" an *NP-property* of a graph?
- Why is the *negation* of "having a perfect matching" an *NP-property* of a bipartite graph?
- Why is the property of "having a perfect matching" in a bipartite graph *well-characterized*?
- What does it mean for *a class of graphs to be in NP*?
- What does it mean for *a class of graphs to be in co-NP*?

1. How do we extend the concept of "a *property* being well-characterized" to that of "an *invariant* being well-characterized"?

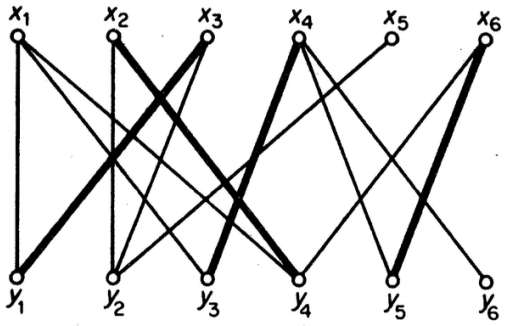
2. What is an example of "an *invariant* being well-characterized"?

3. What is a *minimax theorem*?

4. What is the **importance** of minimax theorems?

5. Let M be a matching in a graph. What is an *M-alternating path*?

6. Let M be a matching in a graph. What is an *M-augmenting path*?



7. Let M be the highlighted lines. Find an M-augmenting path in this graph.

8. What is Berge's Theorem?

9. Why is Berge's Theorem true?

10. What is the *Hungarian Method*?

11. Why does the Hungarian method produce a *provably* maximum matching in a bipartite graph?