**Last name** _____

First name _____

LARSON—MATH 556—CLASSROOM WORKSHEET 08
König, Frobenius and Hall Theorems

Concepts & Notation

- *assignment problem, graph G, points V(G), lines E(G), adjacent, incident.*

- *line covering, line covering number ρ, matching, matching number ν, point covering, point covering number τ, independent set, independence number α.*

Review

- **König's Theorem:** For any bipartite graph, $\tau = \nu$.
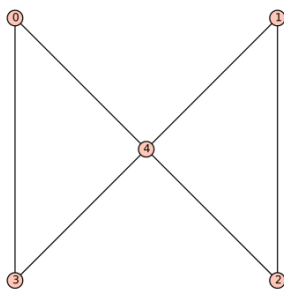
1. Let G be the "bow tie" graph in the picture. Is G bipartite?

2. Find $\nu(G)$ and $\tau(G)$.

The **neighbors** $\Gamma(X)$ of a set of points X is all points in $V(G)$ which are adjacent to at least one point of X.

3. Let $X = \{0, 2\}$ in the bow tie graph. Find $\Gamma(X)$.

4. Let $X = \{0, 4\}$ in the bow tie graph. Find $\Gamma(X)$.

A **perfect matching** (or **1-factor**) is a matching which covers all points of G.

5. Argue that the bow tie graph cannot have a perfect matching.

6. Find a bipartite graph G with a perfect matching.

7. What is *Hall's Theorem*?

8. What is *Frobenius's (Marriage) Theorem*?

9. What is a *Hamilton cycle* in a graph?

10. What's an algorithm for finding a Hamilton cycle in a graph?

11. What is the conceptual difference between the problems (1) of finding a perfect matching in a bipartite graph, and (2) finding a Hamilton cycle in a graph?

12. Why is "having a perfect matching" an *NP-property* of a graph?

13. Why is "having a set of points X where $|X| > |\Gamma(X)|$ an *NP-property* of a graph?

14. Why is the *negation* of "having a perfect matching" an *NP-property* of a bipartite graph?

15. Why is the property of "having a perfect matching" in a bipartite graph *well-characterized*?

16. What does it mean for *a class of graphs to be in NP*?

17. What does it mean for *a class of graphs to be in co-NP*?

18. How do we extend the concept of "a *property* being well-characterized" to that of "an *invariant* being well-characterized"?