Last name ______________________

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LARSON—MATH 556—CLASSROOM WORKSHEET 07
The Proof of König's Minimax Theorem

Concepts & Notation

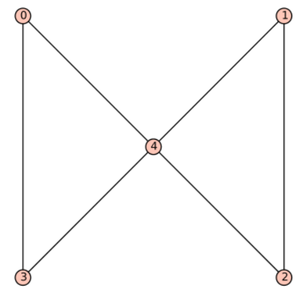
- *assignment problem*, *graph* G, *points* $V(G)$, *lines* $E(G)$, *adjacent*, *incident*.
- *line covering*, *line covering number* ρ, *matching*, *matching number* ν, point covering, *point covering number* τ, *independent set*, *independence number* α.

Review

- If G is a graph and H is also a graph the points and lines of which are also points and lines of G , then H is a **subgraph** of G. If H is a subgraph of G, and if every line joining two points of H which lies in G also lies in H then H is an **induced** subgraph of G. If X is a set of points in G then the **subgraph of G induced by X**, $G[X]$, is the induced subgraph of G having point set X.
- Let C be any set of lines in a graph and let $V(C)$ be the set of points incident to those lines. $\langle C\rangle$ is the subgraph with point set $V(C)$ and lines C.
- Gallai Identity: for any graph G (with no isolated points), $\nu(G) + \rho(G) = |V(G)|$.
- The **symmetric difference** of sets A and B, denoted $A \oplus B$, is $(A \setminus B) \cup (B \setminus A)$.

The Proof of Theorem 1.1.1

1. **Prove:** For any bipartite graph, $\tau = \nu$.



2. Let G be the "bow tie" graph in the picture. Find $\nu(G)$ and $\tau(G)$.

The **neighbors** $\Gamma(X)$ of a set of points X is all points in $V(G)$ which are adjacent to at least one point of X.

3. Let $X = \{0, 2\}$ in the bow tie graph. Find $\Gamma(X)$.

4. Let $X = \{0, 4\}$ in the bow tie graph. Find $\Gamma(X)$.

A **perfect matching** (or **1-factor**) is a matching which covers all points of G.

5. Argue that the bow tie graph cannot have a perfect matching.

6. Find a bipartite graph G with a perfect matching.