Last name ______________________

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LARSON—MATH 556—CLASSROOM WORKSHEET 06
The Proof of König's Minimax Theorem

Concepts & Notation

- *assignment problem*, *graph* G, *points* $V(G)$, *lines* $E(G)$, *adjacent*, *incident*.
- *line covering*, *line covering number* ρ, *matching*, *matching number* ν, point covering, *point covering number* τ, *independent set*, *independence number* α.

Review

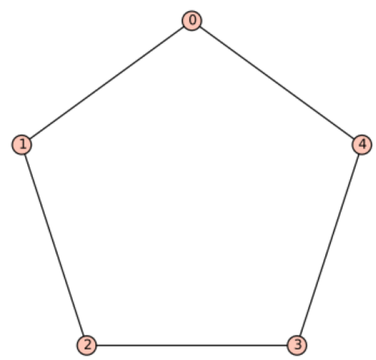
- If G is a graph and H is also a graph the points and lines of which are also points and lines of G , then H is a **subgraph** of G. If H is a subgraph of G, and if every line joining two points of H which lies in G also lies in H then H is an **induced** subgraph of G. If X is a set of points in G then the **subgraph of G induced by X**, $G[X]$, is the induced subgraph of G having point set X.
- Let C be any set of lines in a graph and let $V(C)$ be the set of points incident to those lines. $\langle C\rangle$ is the subgraph with point set $V(C)$ and lines C.
- Gallai Identity: for any graph G (with no isolated points), $\nu(G) + \rho(G) = |V(G)|$.

The Proof of Theorem 1.1.1

1. The **symmetric difference** of sets A and B, denoted $A \oplus B$, is $(A \setminus B) \cup (B \setminus A)$. Let $A = \{a, b, c, d\}$ and $B = \{b, d, e\}$. Find $A \oplus B$.

2. Draw any bipartite graph G with 5 points. Let S be any 3-element subset of $V(G)$. Draw $G[S]$. Check that $G[S]$ is bipartite.

3. Let G be any bipartite graph and $S \subseteq G$. Show (that is, argue, prove): $G[S]$ is bipartite.



4. Let G be this cycle. Find a minimum point cover of G. Find $\tau(G)$.

5. Let G be this cycle and let x be the line $\{0, 1\}$. Draw $G - x$.

6. Find a minimum point cover S_x of $G - x$. Find $\tau(G - x)$.

7. Is it true that for *every* line x of G that $\tau(G - x) = \tau(G) - 1$?

8. Argue that no minimum point cover of $G - x$ can contain either of the points 0 or 1.

9. Let G be the cycle above and let y be the line $\{0, 4\}$. Draw $G - y$.

10. Find a minimum point cover S_y of $G - y$. Find $\tau(G - y)$.

11. Argue that no minimum point cover of $G - y$ can contain either of the points 0 or 4.

12. Find $S_x \oplus S_y$

13. Draw $G[\{0\} \cup (S_x \oplus S_y)]$.

14. **Prove:** For any bipartite graph, $\tau = \nu$.