Last name ______________________

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LARSON—MATH 556—CLASSROOM WORKSHEET 05
The Proof of Lemma 1.02 (a Gallai Identity)

Concepts & Notation

- *assignment problem*, *graph* G, *points* $V(G)$, *lines* $E(G)$, *adjacent*, *incident*.
- *line covering*, *line covering number* ρ, *matching*, *matching number* ν, point covering, *point covering number* τ, *independent set*, *independence number* α.

Review

- If G is a graph and H is also a graph the points and lines of which are also points and lines of G , then H is a **subgraph** of G. If H is a subgraph of G, and if every line joining two points of H which lies in G also lies in H then H is an **induced** subgraph of G. If X is a set of points in G then the **subgraph of G induced by X**, $G[X]$, is the induced subgraph of G having point set X.
- An alternating sequence of points and lines, beginning and ending with points, is called a **walk**. If all lines in a walk are distinct, the walk is called a **trail**. If, in addition, the points are also distinct, the trail is a **path**. A graph is **connected** if every two points are joined by a path. A maximal connected subgraph of a graph G is a **component** of G.
- Let C be any set of lines in a graph and let $V(C)$ be the set of points incident to those lines. $\langle C \rangle$ is the subgraph with point set $V(C)$ and lines C.

The Proof of Lemma 1.02

1. Explain why, for any graph G (with no isolated points), $\nu(G) + \rho(G) = |V(G)|$.

The Proof of Theorem 1.1.1

2. The **symmetric difference** of sets A and B, denoted $A \oplus B$, is $(A \setminus B) \cup (B \setminus A)$. Let $A = \{a, b, c, d\}$ and $B = \{b, d, e\}$. Find $A \oplus B$.

3. Draw any bipartite graph G with 5 points. Let S be any 3-element subset of $V(G)$. Draw $G[S]$. Check that $G[S]$ is bipartite.

4. Let G be any bipartite graph and $S \subseteq G$. Show (that is, argue, prove): $G[S]$ is bipartite.

5. Draw K_4, the complete graph on 4 points. Label the vertices. Remove any line x. Call the resulting graph $K_4 - x$. List the remaining edges $E(K_4 - x)$.

6. Find a maximum matching in $K_4 - x$.

7. Find $\nu(K_4 - x)$.

8. Find a minimum line cover of $K_4 - x$.

9. Find $\rho(K_4 - x)$.

10. Find a minimum point cover of $K_4 - x$.

11. Find $\tau(K_4 - x)$.

12. Find a maximum independent set of $K_4 - x$.

13. Find $\alpha(K_4 - x)$.