Last name ______________________

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LARSON—MATH 556—CLASSROOM WORKSHEET 04
The Proof of Lemma 1.02 (a Gallai Identity)

Concepts & Notation

- *assignment problem*, *graph* G, *points* $V(G)$, *lines* $E(G)$, *adjacent*, *incident*.
- *line covering*, *line covering number* ρ, *matching*, *matching number* ν, point covering, *point covering number* τ, *independent set*, *independence number* α.

Review

- A **star** is the complete bipartite graph $K_{1,n}$. ($K_{1,n}$ is the n-star, or star on $n+1$ points).
- A **matching** in a graph is a set of lines where no pair of lines is incident to the same point. A matching is **maximum** if there is no matching with larger cardinality. The cardinality of a maximum matching in graph G is the **matching number** $\nu(G)$.
- A **line cover** in a graph is a set of lines which are incident to all the points in the graph. A line cover is *minimum* if there is no line cover with small cardinality. The cardinality of a minimum line cover in graph G is the *line covering number* $\rho(G)$.
- An *independent set* in a graph is a set of points no pair of which are adjacent. An independent set is **maximum** if there is no independent set with larger cardinality. The cardinality of a maximum independent set in graph G is the *independence number* $\alpha(G)$.

Basic Definitions & the 4 fundamental invariants in Chp. 1

1. Find a maximum independent set for $K_{3,4}$. Then find α. Can you find an independent set which is maxim**al** (can't be extended) but is not maxim*um*?

   A **point cover** is a set of points which are incident to all the lines in the graph. A point cover is *minimum* if there is no point cover with smaller cardinality. The cardinality of a minimum matching in graph G is the **point covering number** $\tau(G)$.

2. Find a minimum point cover for $K_{3,4}$. Then find τ. Can you find a point cover which is minim*al* (can't be reduced) but is not minim*um*?

The Proof of Lemma 1.02

3. What relationship do you notice about ρ, ν and $|V(G)|$ in $K_{3,4}$? Draw another graph and see if this relationship holds. Can you draw a graph where it doesn't hold?

If G is a graph and H is also a graph the points and lines of which are also points and lines of G , then H is a **subgraph** of G. If H is a subgraph of G, and if every line joining two points of H which lies in G also lies in H then H is an **induced** subgraph of G. If X is a set of points in G then the **subgraph of G induced by X**, $G[X]$, is the induced subgraph of G having point set X.

4. Find a subgraph of $K_{3,4}$ which is not an induced subgraph of $K_{3,4}$. What can you say about any induced subgraph of $K_{3,4}$?

An alternating sequence of points and lines, beginning and ending with points, is called a **walk**. If all lines in a walk are distinct, the walk is called a **trail**. If, in addition, the points are also distinct, the trail is a **path**. A graph is **connected** if every two points are joined by a path. A maximal connected subgraph of a graph G is a **component** of G.

5. Draw a graph with 3 components.

6. Draw a connected graph with a line so that removing the line makes the graph dis-connected.

7. Draw a connected graph where removing an line makes the graph dis-connected.

8. What is the fewest number of lines that need to be removed from $K_{3,4}$ in order to make it dis-connected?

Let C be any set of lines in a graph and let $V(C)$ be the set of points incident to those lines. $\langle C\rangle$ is the subgraph with point set $V(C)$ and lines C.

9. Let be C is a minimum line cover of a graph. What must $\langle C\rangle$ "look like"? Draw some examples. Will $\langle C\rangle$ be connected?

10. Explain why, for any graph G (with no isolated points), $\nu(G) + \rho(G) = |V(G)|$.