Last name ______________________

First name ______________________

LARSON—MATH 556—CLASSROOM WORKSHEET 02
The Assignment Problem & Matchings.

Concepts & Notation

- *assignment problem*, *graph* G, *points* $V(G)$, *lines* $E(G)$, *adjacent*, *incident*.

Review

A **graph** G consists of a set of **points** $V(G)$ and **lines** $E(G)$ which are pairs $\{v, w\} \subseteq V(G)$ or, simply, vw. If vw is a line, point v is said to be **adjacent** to point w, while v is **incident** to line vw.

Problems

A **bipartite graph** is a graph G where the points $V(G)$ can be partitioned into two sets A and B such that every edge has one endpoint in A and the other in B. In the assignment problem, the corresponding graph is bipartite: let A be the set of employees and B be the set of jobs.

1. To define a graph on a set of points, it is enough to define an *adjacency relation* which specifies which vertices are adjacent. $V(G) = \{1, 2, 3, 4, 5\}$. Let a be adjacent to b in G if and only if $a + b$ is even. Is G bipartite?

2. Let A be any set and B be the set of subsets of A. Let $V(G) = A \cup B$, and let $a \in A$ be adjacent to $b \in B$ if and only if $a \in b$. Let $A = \{a_1, a_2\}$. Find B and then draw G. Is G bipartite?

3. Let A be any set and B be the set of subsets of A. Let $V(G) = A \cup B$, and let $a \in A$ be adjacent to $b \in B$ if and only if $a \in b$. Let $A = \{a_1, a_2, a_3\}$. Find B and then draw G. Is G bipartite?

Basic Definitions & the 4 fundamental invariants in Chp. 1

A **bipartite graph** is a graph $G = (A, B)$ with bipartition $V(G) = A \cup B$ where every line has one endpoint in A and the other in B.

A **complete bipartite graph** is a graph $G = (A, B)$ with bipartition $V(G) = A \cup B$ where there is a line between every point in A and point in B. $K_{m,n}$ denotes the complete bipartite graph with $|A| = n$ and $|B| = m$.

4. Draw the complete bipartite graph $K_{3,3}$.

5. Draw the complete bipartite graph $K_{3,4}$.

A **star** is the complete bipartite graph $K_{1,n}$. ($K_{1,n}$ is the n-star, or star on $n+1$ points).

6. Draw the star $K_{1,3}$.

7. Draw the 2-star and the 6-star.

A **matching** in a graph is a set of lines where no pair of lines is incident to the same point. A matching is **maximum** if there is no matching with larger cardinality. The cardinality of a maximum matching in graph G is the **matching number** $\nu(G)$.

8. Find a maximum matching for $K_{3,4}$. Then find ν. Can you find a matching which is maxim*al* (can't be extended) but is not maxim*um*?

A **line cover** in a graph is a set of lines which are incident to all the points in the graph. A line cover is *minimum* if there is no line cover with small cardinality. The cardinality of a minimum line cover in graph G is the *line covering number* $\rho(G)$.

9. Find a minimum line cover for $K_{3,4}$. Then find ρ. Can you find a line cover which is minim**al** (can't be reduced) but is not minim*um*?

An *independent set* in a graph is a set of points no pair of which are adjacent. An independent set is **maximum** if there is no independent set with larger cardinality. The cardinality of a maximum independent set in graph G is the *independence number* $\alpha(G)$.

10. Find a maximum independent set for $K_{3,4}$. Then find α. Can you find an independent set which is maxim**al** (can't be extended) but is not maxim*um*?

A **point cover** is a set of points which are incident to all the lines in the graph. A point cover is *minimum* if there is no point cover with smaller cardinality. The cardinality of a minimum matching in graph G is the **point covering number** $\tau(G)$.

11. Find a minimum point cover for $K_{3,4}$. Then find τ. Can you find a point cover which is minim*al* (can't be reduced) but is not minim*um*?

The Proof of Lemma 1.02

12. What relationship do you notice about ρ, ν and $|V(G)|$ in $K_{3,4}$? Draw another graph and see if this relationship holds. Can you draw a graph where it doesn't hold?

If G is a graph and H is also a graph the points and lines of which are also points and lines of G , then H is a **subgraph** of G. If H is a subgraph of G, and if every line joining two points of H which lies in G also lies in H then H is an **induced** subgraph of G. If X is a set of points in G then the **subgraph of G induced by X**, $G[X]$, is the induced subgraph of G having point set X.

13. Find a subgraph of $K_{3,4}$ which is not an induced subgraph of $K_{3,4}$. What can you say about any induced subgraph of $K_{3,4}$?