Last name _____

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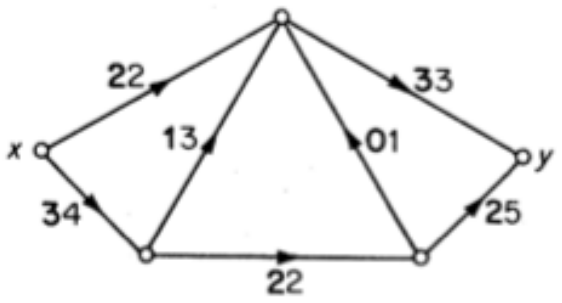
LARSON—MATH 356—HOMEWORK WORKSHEET 13
Network Flows



In this network $X = (D, x, y, cap)$, x is the *source* and y is the *sink*. Label the remaining vertices anything. There are two numbers on each edge: the first number is the current flow f and the second number is the edge *capacity*.

1. Check that f is a *flow*. Explain.
2. What is the *value* of f? Explain.

```
procedure scan(u:vertex;X :network; f:flow );
for every 'unlabeled' vertex v that is connected
    to u by an edge in either or both directions, do
      if the flow in (u, v) is less than cap(u, v)
          then
        label v with (u, +, min{z(u), cap(u, v) − flow(u, v)})
          else if the flow in (v, u) is > 0
                then
            label v with (u, −, min{z(u), flow(v, u)}) and
            change the label-status of v to 'labeled';
change the scan-status of u to 'scanned'
end.{scan}
```

3. We will find a flow-augmenting path in the network X with specified flow f. To begin no vertices are labeled or scanned. Initially label the sink x with $(-\infty, +, \infty)$. Now scan the sink x. What do we get?
4. What is the main idea of the vertex-scanning algorithm?
5. Create a table of the vertices, the labels you create, and whether or not the vertex has been scanned. When you label a vertex, explain what the 3 parts of the label mean.

```
procedure labelandscan(X :network; f:flow; whyhalt:reason);
give every vertex the scan-status 'unscanned'
      and the label-status 'unlabeled';
u := source;
label source with (−∞, +, ∞);
label-status of source:= 'labeled';
while {there is a 'labeled' and 'unscanned' vertex v
                and sink is 'unlabeled'}
                    do scan(v, X, f);
  if sink is unlabeled
        then 'whyhalt':='flow is maximum'
        else 'whyhalt':= 'it's time to augment'
end.{labelandscan}
```

6. What is the main idea of the "label-and-scan" algorithm?

7. When you reach the sink y (which you must as the indicated flow is not optimal), use the vertex labels to reconstruct a flow-augmenting path P. What is this path? How mush more flow can you "push" across P in the network?

8. Redraw the network and define an improved flow f'. Check that f' is a valid flow. Explain.

9. Now use the "label-and-scan" algorithm until it terminates on your network with new flow f'. We will try to improve the given flow.

10. If you do not find a flow-augmenting path, you will have a final collection of scanned vertices S. What are they? It must be the case that no neighbor of any vertex in S is labelled but not scanned. Explain. What does this mean?