

Last name _____

First name _____

LARSON—MATH 356—HOMEWORK WORKSHEET 07
Test 1 REVIEW.

Definitions. Write the definition **and** an example.

1. What is a *graph* G (formal definition)?
2. What is $V(G)$, $E(G)$?
3. What does it mean for vertices to be *adjacent*?
4. What is an *independent set* of vertices?
5. What is $\maxset(G)$?
6. What is a *subset* of a set?
7. What is a *weighted graph*?
8. What is a *shortest path* between two vertices in a weighted graph?
9. (**little-o**). What does $f(x) = o(g(x))$ mean?
10. (**big-O**). What does $f(x) = O(g(x))$ mean?
11. (**twiddles**). What does $f(x) \sim g(x)$ mean?
12. What is a *first-order* recurrence relation?
13. (**Notation**) What is $[n]$?
14. (**Notation**) If S is a set, what is $|S|$? What is $||[n]||$?
15. What is the *binomial coefficient* $\binom{n}{k}$?
16. What is the *binomial theorem*?
17. What is the *degree* $\rho(v)$ of a vertex v ?
18. What is a *subgraph* of a graph?
19. What is an *induced subgraph* of a graph?

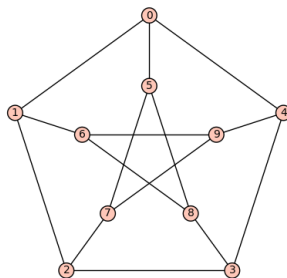
Algorithms

20. Describe an algorithm to find a maximum independent set in a graph?
21. What is *Dijkstra's algorithm*?

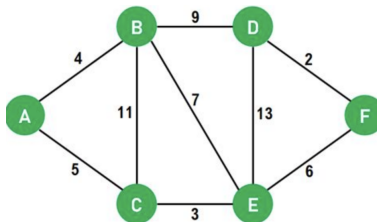
Theorems. Explain why the statement is true.

22. Why does $\sum_{k=0}^n \binom{n}{k} = 2^n$?
23. Why does $\sum_{v \in V(G)} \rho(v) = 2|E(G)|$?

Problems. Explain your answers.



24. Find a maximum independent set I in the Petersen graph. *Argue* that your set I is maximum (that there *can't* be a larger independent set).



25. Use Dijkstra's algorithm to find a shortest path from C to D .
26. Explain why e^x grows *faster* than x^α for any positive α .
27. Explain why x^α grows *faster* than $\log x$ for any positive α .
28. True or False. Explain.
 - (a) $x^2 = o(x^5)$.
 - (b) $1/x = o(1)$.
 - (c) $2 \sin x = O(x)$.
 - (d) $\sin x = O(1)$.
 - (e) $x^2 + x \sim x^2$.
 - (f) $2^x + 7 \log x + \cos x \sim 2^x$.
29. What does $n = (10100)_2$ mean? Find n .
30. Find the base-2 representation of $n = 111$.
31. How many *bits* are in the base-2 representation of an integer n ?
32. Why does $\frac{1 - x^n}{1 - x} = 1 + x + x^2 + \dots + x^{n-1}$?
33. Solve: $x_{n+1} = 5x_n + 7$ ($n \geq 0$; $x_0 = 0$).