Last name ______________________

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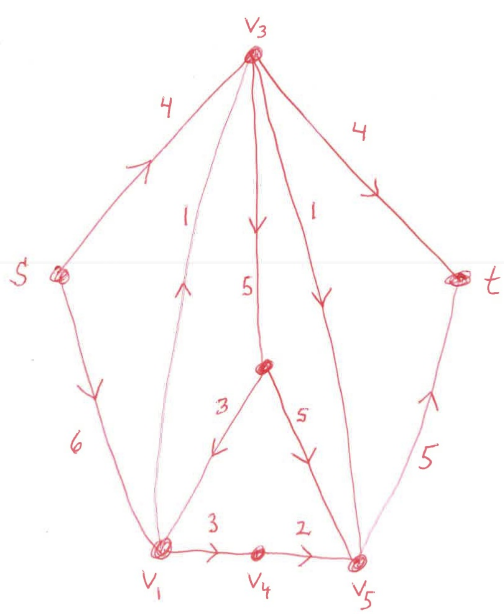
LARSON—MATH 356—CLASSROOM WORKSHEET 25
Network Flows

Review

- What is a *flow* in a network?
- What is the *value* of a flow in a network?
- Given a path P from s to t in X, when is an edge of P *coherent* (and when is it *incoherent*) with respect to P?
- What is a *flow augmenting path*?
- How can you find a flow augmenting path?
- What do the labels $(u, \pm, z)$ mean in Wilf's algorithm?
- What is a "labeled" vertex in Wilf's description of the algorithm for finding a flow-augmenting path. What is a "scanned" vertex?
- What is the main idea of the vertex-scanning algorithm?
- What is the main idea of the "label-and-scan" algorithm?
- What is the termination condition for this algorithm (when will it stop)? What does it mean when this algorithm terminates?



1. Let f be the zero-flow in the above network. Use the label-scan algorithm to find a flow augmenting path in the above network. Repeat until no improvement is possible. What is the value of the final flow?

2. What is a *cut* in a network?

3. What is the *capacity* of a cut in a network?

4. What is the set of scanned vertices in the final iteration of our work finding a maximum flow?

5. What is the corresponding cut?

6. What is the capacity of this cut?

7. Check that the value of our final flow equals the capacity of this cut? (The Max Flow Min Cut Theorem says this is always the case).

8. The main tool we need in arguing that the value of a maximum flow in a network is the minimum capacity of a cut is the following lemma.

Lemma 3.4.1. *Let f be a flow of value Q in a network $\mathbf{X}$, and let $(W, \overline{W})$ be a cut in $\mathbf{X}$. Then*

$$Q = f(W, \overline{W}) - f(\overline{W}, W) \leq cap(W, \overline{W}). \tag{3.4.1}$$

Proof of lemma: The net flow out of s is Q. The net flow out of any other vertex $w \in W$ is 0. Hence, if $V(\mathbf{X})$ denotes the vertex set of the network $\mathbf{X}$, we obtain

$$\begin{aligned}
Q &= \sum_{w \in W} \{f(w, V(\mathbf{X})) - f(V(\mathbf{X}), w)\} \\
&= f(W, V(\mathbf{X})) - f(V(\mathbf{X}), W) \\
&= f(W, W \cup \overline{W}) - f(W \cup \overline{W}, W) \\
&= f(W, W) + f(W, \overline{W}) - f(W, W) - f(\overline{W}, W) \\
&= f(W, \overline{W}) - f(\overline{W}, W).
\end{aligned}$$