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LARSON—MATH 356—CLASSROOM WORKSHEET 23
Network Flows

Review

- What is a *flow* in a network?
- What is the *value* of a flow in a network?
- Given a path P from s to t in X , when is an edge of P *coherent* (and when is it *incoherent*) with respect to P ?

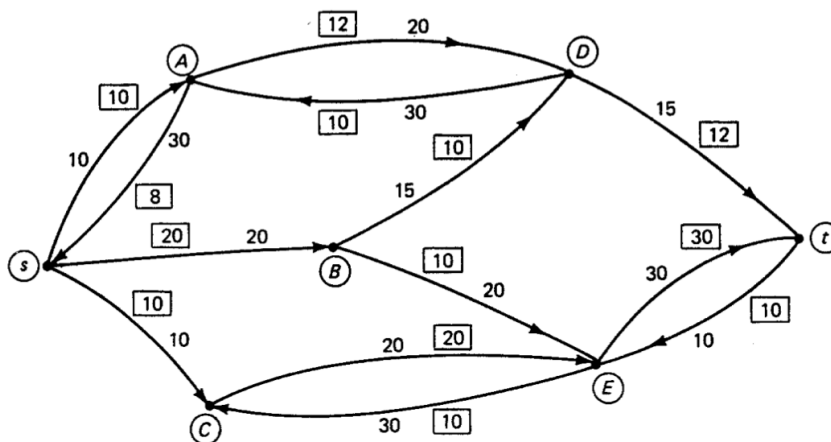


Fig. 3.1.2: A flow in a network

1. What is a *flow augmenting path*?
2. How can you find a flow augmenting path?
3. What do the labels $(u, \pm, z)$ mean in Wilf's algorithm?
4. What is a "labeled" vertex in Wilf's description of the algorithm for finding a flow-augmenting path. What is a "scanned" vertex?

5. What is the main idea of the vertex-scanning algorithm?

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procedure scan(u:vertex; X:network; f:flow);
for every 'unlabeled' vertex v that is connected
  to u by an edge in either or both directions, do
    if the flow in (u, v) is less than cap(u, v)
      then
        label v with (u, +, min{z(u), cap(u, v) - flow(u, v)})
      else if the flow in (v, u) is > 0
        then
          label v with (u, -, min{z(u), flow(v, u)}) and
          change the label-status of v to 'labeled';
change the scan-status of u to 'scanned'
end.{scan}

```

6. We will find a flow-augmenting path in the above network with the specified flow. To begin no vertices are labeled or scanned. Initially label the sink *s* with $(-\infty, +, \infty)$. Scan the sink *s*. What do we get?

7. What is the main idea of the “label-and-scan” algorithm?

8. What is the termination condition for this algorithm (when will it stop)? What does it mean when this algorithm terminates?

```

procedure labelandscan(X:network; f:flow; whyhalt:reason);
give every vertex the scan-status 'unscanned'
and the label-status 'unlabeled';
u := source;
label source with  $(-\infty, +, \infty)$ ;
label-status of source := 'labeled';
while {there is a 'labeled' and 'unscanned' vertex v
  and sink is 'unlabeled'}
  do scan(v, X, f);
  if sink is unlabeled
    then 'whyhalt' := 'flow is maximum'
    else 'whyhalt' := 'it's time to augment'
  end.{labelandscan}

```

9. Now use the “label-and-scan” algorithm until it terminates. We will try to improve the given flow.
10. When the algorithm terminates we can use the vertex labels to find a flow-augmenting path. What is this flow-augmenting path.
11. Use the flow augmenting path to define a new flow f' . What is the value of the new flow?