Last name _____

First name _____

LARSON—MATH 356—CLASSROOM WORKSHEET 11
Recurrence Relations

Review

- How does Wilf use differentiation and reference to the series zoo to evaluate:

$$1 + 2 \cdot 2 + 3 \cdot 4 + 4 \cdot 8 + 5 \cdot 16 + \ldots N \cdot 2^{N-1}?$$

- How does Wilf use reference to the series zoo and manipulation to evaluate:

$$\frac{1}{2 \cdot 3^2} + \frac{1}{3 \cdot 3^3} + \ldots?$$

- What is a *first-order* recurrence relation?

Steps to solve $x_{n+1} = b_{n+1}x_n + c_{n+1}$ $(n \geq 1;\ x_0$ given).

- Let a new sequence $y_1, y_2, \ldots$ be defined by $x_n = b_1 b_2 \ldots b_n y_n$ $(n \geq 1;\ x_0 = y_0)$.

- Substitute for x_n in the original recurrence.

- Divide by the coefficients of the y's to get $y_{n+1} = y_n + d_{n+1}$. $(n \geq 0,\ y_0$ given), with $d_{n+1} = \frac{c_{n+1}}{b_1 \ldots b_{n+1}}$.

- So $y_n = y_0 + \sum_{j=1}^{n} d_j$ $(n \geq 0)$. Now reverse the change of variables.

1. Solve: $x_{n+1} = 3x_n + n$ $(n \geq 0;\ x_0 = 0)$.

2. What is a *second-order* recurrence relation?

3. How does Wilf use the "geometric series" idea to solve the Fibonacci sequence recurrence?

Counting

4. (**Notation**)What is $[n]$?

5. (**Notation**) If S is a set, what is $|S|$? What is $|[n]|$?

6. What is the *binomial coefficient* $\binom{n}{k}$?

7. Why does $\binom{n}{k} = \binom{n}{n-k}$?

8. What is *Pascal's Triangle*?

9. How many subsets does an n-element set have?

10. Why does $\sum_{k=0}^{n} \binom{n}{k} = 2^n$?

11. What is the *binomial theorem*?