Last name ________________________

First name ________________________

LARSON—MATH 356—CLASSROOM WORKSHEET 10
Manipulations with Series & Recurrence Relations

Review

- Why does $\dfrac{1-x^n}{1-x} = 1 + x + x^2 + \ldots + x^{n-1}$?

$$\sum_{k=0}^{\infty} x^k = 1/(1-x) \qquad (|x|<1)$$

$$e^x = \sum_{m=0}^{\infty} x^m/m!$$

$$\sin x = \sum_{r=0}^{\infty} (-1)^r x^{2r+1}/(2r+1)!$$

$$\cos x = \sum_{s=0}^{\infty} (-1)^s x^{2s}/(2s)!$$

$$\log(1/(1-x)) = \sum_{j=1}^{\infty} x^j/j \qquad (|x|<1)$$

Manipulations with Series, and "the Zoo"

1. How does Wilf use differentiation and reference to the series zoo to evaluate:

$$1 + 2\cdot 2 + 3\cdot 4 + 4\cdot 8 + 5\cdot 16 + \ldots N\cdot 2^{N-1}?$$

2. How does Wilf use reference to the series zoo and manipulation to evaluate:

$$\frac{1}{2\cdot 3^2} + \frac{1}{3\cdot 3^3} + \ldots?$$

3. Find an explicit formula for:

$$\sum_{m>1} \frac{(2m+7)}{5^m}.$$

Recurrence Relations

4. What is a *first-order* recurrence relation?

Steps to solve $x_{n+1} = b_{n+1}x_n + c_{n+1}$ $(n \geq 1;\ x_0 \text{ given})$.

- Let a new sequence $y_1, y_2, \ldots$ be defined by $x_n = b_1 b_2 \ldots b_n y_n$ $(n \geq 1;\ x_0 = y_0)$.
- Substitute for x_n in the original recurrence.
- Divide by the coefficients of the y's to get $y_{n+1} = y_n + d_{n+1}$. $(n \geq 0,\ y_0 \text{ given})$, with $d_{n+1} = \frac{c_{n+1}}{b_1 \ldots b_{n+1}}$.
- So $y_n = y_0 + \sum_{j=1}^{n} d_j$ $(n \geq 0)$. Now reverse the change of variables.

5. Solve: $x_{n+1} = 3x_n + n$ $(n \geq 0;\ x_0 = 0)$.

6. What is a *second-order* recurrence relation?

7. How does Wilf use the "geometric series" idea to solve the Fibonacci sequence recurrence?