Last name _____

First name _____

LARSON—MATH 356—CLASSROOM WORKSHEET 09
Integer representations & Manipulations with Series

Review

- What does $n = (12345)_{10}$ mean? Find n.
- What does $n = (10100)_2$ mean? Find n.
- Find the base-2 representation of $n = 17$.
- Find the base-2 representation of $n = 111$.
- **Claim:** Any integer n can be be written uniquely as $n = b_0 \cdot 2^0 + b_1 \cdot 2^1 + \ldots + b_k \cdot 2^k$, where $b_i \in \{0, 1\}$ and $b_k > 0$.

Integer Representations

1. How many *bits* are in the base-2 representation of an integer n?

Manipulations with Series, and "the Zoo"

2. Why does $\dfrac{1 - x^n}{1 - x} = 1 + x + x^2 + \ldots + x^{n-1}$?

3. Evaluate $\sum_{j=0}^{9} 3^j$.

$$\sum_{k=0}^{\infty} x^k = 1/(1-x) \qquad (|x| < 1)$$

$$e^x = \sum_{m=0}^{\infty} x^m/m!$$

$$\sin x = \sum_{r=0}^{\infty} (-1)^r x^{2r+1}/(2r+1)!$$

$$\cos x = \sum_{s=0}^{\infty} (-1)^s x^{2s}/(2s)!$$

$$\log(1/(1-x)) = \sum_{j=1}^{\infty} x^j/j \qquad (|x| < 1)$$

4. Find $1 + \log 2 + \dfrac{(\log 2)^2}{2!} + \dfrac{(\log 2)^3}{3!} + \ldots$.

5. How does Wilf use differentiation and reference to the series zoo to evaluate:
$$1 + 2 \cdot 2 + 3 \cdot 4 + 4 \cdot 8 + 5 \cdot 16 + \ldots N \cdot 2^{N-1}?$$

6. How does Wilf use reference to the series zoo and manipulation to evaluate:
$$\frac{1}{2 \cdot 3^2} + \frac{1}{3 \cdot 3^3} + \ldots?$$

7. Find an explicit formula for:
$$\sum_{m>1} \frac{(2m+7)}{5^m}.$$