Last name ______________________

First name ______________________

LARSON—MATH 356—CLASSROOM WORKSHEET 08
Orders of Magnitude & Integer representations

Orders of Magnitude

Definition (twiddles). We say that $f(x) \sim g(x)$ if $\lim_{x\to\infty} \dfrac{f(x)}{g(x)} = 1$.

1. (a) $x^2 + x \sim x^2$.

   (b) $(3x+1)^4 \sim 81x^4$.

   (c) $\sin \frac{1}{x} \sim \frac{1}{x}$.

   (d) $\frac{(2x^3+5x+7)}{(x^2+4)} \sim 2x$.

   (e) $2^x + 7\log x + \cos x \sim 2^x$.

The last symbol in the asymptotic set that we will need is the 'Ω.' In a nutshell, 'Ω' is the negation of 'o.' That is to say, $f(x) = \Omega(g(x))$ means that it is not true that $f(x) = o(g(x))$. In the study of algorithms for computers, the 'Ω' is used when we want to express the thought that a certain calculation takes at least so-and-so long to do.

Definition (Omega). We say that $f(x) = \Omega(g(x))$ if there is an $\epsilon > 0$ and a sequence $x_1, x_2, x_3, \ldots \to \infty$ such that $\forall j : |f(x_j)| > \epsilon g(x_j)$.

2. Show $2^x = \Omega(x^\alpha)$ (for any $\alpha > 0$).

Definition. A function f is of **exponential growth** if there exists $c > 1$ such that $f(x) = \Omega(c^x)$ and there exists d such that $f(x) = O(d^x)$.

3. Show that $f(x) = 2^x$ grows exponentially according to this definition.

4. How about $f(x) = 2^x + x^{100}$?

5. What does $n = (12345)_{10}$ mean? Find n.

6. What does $n = (10100)_2$ mean? Find n.

7. Find the base-2 representation of $n = 17$.

8. Find the base-2 representation of $n = 111$.

9. **Claim:** Any integer n can be be written uniquely as $n = b_0 \cdot 2^0 + b_1 \cdot 2^1 + \ldots + b_k \cdot 2^k$, where $b_i \in \{0, 1\}$ and $b_k > 0$.

10. How many *bits* are in the base-2 representation of an integer n?