Last name ______________________

First name ______________________

LARSON—MATH 356—CLASSROOM WORKSHEET 06
Orders of Magnitude

Review

- How "fast" is Dijkstra's algorithm? What can we say here?
- Why does e^x grow "faster" than any positive power of x?

Orders of Magnitude

1. Explain why 2^x grows *faster* than x^α for any positive α.

2. Explain why x^α grows *faster* than $\log x$ for any positive α.

Definition (little-o). We say that $f(x) = o(g(x))$ $(x \to \infty)$ if $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ exists and is equal to 0.

3. Check these examples:

   (a) $x^2 = o(x^5)$.

   (b) $\sin x = o(x)$.

   (c) $14.709\sqrt{x} = o(\frac{x}{2} + 7\cos x)$.

   (d) $1/x = o(1)$.

   (e) $23 \log x = o(x^{.02})$

Definition (big-O). We say that $f(x) = O(g(x))$ $(x \to \infty)$ if $\exists C, x_0$ such that $|f(x)| < Cg(x)$ $(\forall x > x_0)$.

4. Check these examples:
   (a) $\sin x = O(x)$.

   (b) $\sin x = O(1)$.

   (c) $x^3 + 5x^2 + 77\cos x = O(x^5)$.

   (d) $\frac{1}{(1+x^2)} = O(1)$.

Definition (Theta). We say that $f(x) = \Theta(g(x))$ if there are constants $c1 > 0$, $c2 > 0$, x_0 such that for all $x > x_0$, it is true that $c_1 g(x) < f(x) < c_2 g(x)$.

5. Check these examples:
   (a) $(x+1)^2 = \Theta(3x^2)$.

   (b) $\frac{(x^2+5x+7)}{(5x^3+7x+2)} = \Theta(\frac{1}{x})$.

   (c) $\sqrt{3+\sqrt{2x}} = \Theta(x^{\frac{1}{4}})$.

   (d) $(1+\frac{3}{x})^x = \Theta(1)$.

Definition (twiddles). We say that $f(x) \sim g(x)$ if $\lim_{x\to\infty} \dfrac{f(x)}{g(x)} = 1$.

6. What are some examples?