Last name ______________________

First name ______________________

LARSON—MATH 353–HOMEWORK WORKSHEET 10

Suggestions: Write out lots of examples. Collect evidence. Doodle. You won't sit down knowing the right idea. But it **will** come if you start early, wrestle with the problem, read, sleep on it, and come back to it.

From Stein—Chapter 2

1. For what values of n is $\phi(n)$ odd?

2. Show that if p is a positive integer such that both p and $p^2 + 2$ are prime, then $p = 3$

3. What is a *primitive root* in $\mathbb{Z}/n\mathbb{Z}$ (for positive integer n)?

4. For each n from $n = 3$ to $n = 9$, find all the primitive roots (explain your steps).

5. We saw numerical evidence that $x = n^2 + 1$ primes (for $x > 5 = 2^2 + 1$, written in base-10 (that is, decimal digits), must end in 1 or 7. Is this true? If so, can you think of an argument.

6. **Set up** the following problem (what kind of problem is this?):

   Seven competitive math students try to share a huge hoard of stolen math books equally between themselves. Unfortunately, six books are left over, and in the fight over them, one math student is expelled. The remaining six math students, still unable to share the math books equally since two are left over, again fight, and another is expelled. When the remaining five share the books, one book is left over, and it is only after yet another math student is expelled that an equal sharing is possible. What is the minimum number of books that allows this to happen?.