Last name _____

First name _____

LARSON—MATH 353–HOMEWORK WORKSHEET 08
Conjecturing in CoCalc/SAGE.

Setup

1. Start the Chrome browser.
2. Go to `https://cocalc.com`
3. Log in to your account.
4. You should see an existing Project for our class. Click on that.
5. Make sure you are in your Home directory (if you put files in the Handouts directory they could be overwritten.)
6. Click "New", then "Jupyter Notebook", then call it **353-h08**.
7. Make sure you have SAGE as the *kernel*.
8. Check that you have a `conjecturing.py` file and an `expressions` file in your `Home` directory.
9. Copy the most recent version of `number_theory.sage` from the Handouts directory to your Home directory (if you have been adding your own notes and code to that copy, rename it first so it doesn't get copied over).

Investigation

1. Start with the following initial run for a **lower-bound** for `count_prime_divisors`, using $x = n^2 + 1$ integers as the data/objects/input, and where we will interpret produced conjectures as being true for these integers. (Note that the theorems we proved in class are not recorded here—those are proved upper bounds).

   What conjectures do you get?

```
objects = [5,17,65,901,325,170,2210]

invariants = [digits10, digits2, count_divisors, count_prime_divisors,
number, euler_phi, sigma, base, count_divisors_base,
    count_prime_divisors_base, euler_phi_base, sigma_base]

theorems = []

inv_of_interest = invariants.index(count_prime_divisors)

conjs = conjecture(objects, invariants, inv_of_interest, upperBound =
    False, theory = theorems, debug = True)

for conj in conjs:
    print(conj)
```

2. For each produced conjecture, test whether it is true for all the $x = n^2+1$ integers in the $Sp1$ list. If you find a counterexample, report the smallest integer which is a counterexample.

3. If you found any counterexamples, add these to your `objects` list, and then re-run the conjecturing program (do that in a new cell so you have a full history of your investigations.

4. When you have a run of conjectures, all of which are true for all the $Sp1$ integers, then choose a conjecture that interests you, write the conjecture and all relevant definitions in a new cell.

5. Can you prove it? If so, add it as a theorem and generate new conjectures.

6. Read the Wikipedia number theory page, or the Sagemath number theory page, or find a research article or book with a number theory invariant that we are not currently using. If its not built-in you'll have to code it, or ask ChatGPT to code it, or see me and get help. Explain your invariant and give some examples. (And give a reference for your source).

7. What is your code for this invariant? Test it to make sure that it works.

8. Add it to your list of invariants (again, make a new cell, so you have your entire complete lab record) and rerun the conjecturing program. Do you get anything new (it will only be new if one of the conjectures includes your invariant. If not, they will be the same conjectures as before)?

Facts about the Produced Conjectures

(1) **Truth**. They are TRUE for every input object.

(2) **Significance**. Each conjecture, when added to the list of conjectures, was "better" for at least one input object than any previously stored conjecture.

Getting your homework recorded
When you are done,...

1. Click the "Print" menu choice (under "File") and make a pdf of this worksheet (html is OK too).

2. Send me an email (`clarson@vcu.edu`) with an informative header like
"Math 353 - h08 worksheet attached" (so that it will be properly recorded).

3. Remember to attach your worksheet!