Last name _____

First name _____

LARSON—MATH 353–HOMEWORK WORKSHEET 05
Conjecturing in CoCalc/SAGE.

1. Sign in to your CoCalc account.
   (a) Start the Chrome browser.
   (b) Go to `https://cocalc.com`
   (c) Log in to your account.
   (d) You should see an existing Project for our class. Click on that.
   (e) Make sure you are in your Home directory (if you put files in the Handouts directory they could be overwritten.)
   (f) Click "New", then "Jupyter Notebook", then call it **353-h05**.
   (g) Make sure you have SAGE as the *kernel*.

Review: setting up `Conjecturing`

   (a) We set up the conjecturing program by following the steps at: `http://nvcleemp.github.io/conjecturing/`
   (b) Check that you have a `conjecturing.py` file and an `expressions` file in your `Home` directory.

Facts about the Produced Conjectures

(1) **Truth**. They are TRUE for every input object.
(2) **Significance**. Each conjecture, when added to the list of conjectures, was "better" for at least one input object than any previously stored conjecture.

The **motivation** for this exercise is to practice using the `Conjecturing` program, and to demonstrate that it can be used for *any* kinds of objects which can be represented to a computer. Here we will use graphs as an example.

For background, read the `Wikipedia` page on *graph theory*: `https://en.wikipedia.org/wiki/Graph_theory`.

2. Make a text cell in your Jupyter notebook. Explain what a *complete graph* is.
3. Code and run.

```
K5 = graphs.CompleteGraph(5)
K5.show()
```

4. Define $K4$ to be the complete graph with 4 vertices and make a picture of it.
5. Define $K3$ to be the complete graph with 3 vertices and make a picture of it.

6. What is the *order* of a graph? Make a new text cell in your Jupyter notebook and explain. What is the order of $K3$, $K4$, $K5$?

7. What is the *size* of a graph? Make a new text cell in your Jupyter notebook and explain. What is the size of $K3$, $K4$, $K5$?

8. What is the *degree* of a vertex? Make a new text cell in your Jupyter notebook and explain. What is the degree of every vertex of $K3$, $K4$, $K5$?

9. Code and test each of the following procedures.

```
def size(g):
    return g.size()

def order(n):
    return g.order()

def max_degree(g):
    return max(g.degree())
```

10. Here is a minimal test for generating upper-bound conjectures for a the size of a graph.

```
invariants = [size, order, max_degree]
objects = [K3, K4, K5]
invariant_of_interest = invariants.index(size)
conjectures = conjecture(objects, invariants, invariant_of_interest,
   upperBound = True, debug = True)
```

    If a conjecture is true, the only way to be certain is to *prove* it. If it is false, the only way to be certain of that is to find an example that demonstrates falsity (a *counterexample*).

11. What conjectures do you get? (Are they true? If not find a counterexample and add it to Sage. Then re-run to get new conjectures.)

12. We can get better conjectures by adding more graph invariants. What can we add?

13. Now generate lower-bound conjectures for the size of a graph.

Getting your classwork recorded

When you are done, before you leave class...

(a) Click the "Print" menu choice (under "File") and make a pdf of this worksheet (html is OK too).

(b) Send me an email (`clarson@vcu.edu`) with an informative header like "Math 353 - h05 worksheet attached" (so that it will be properly recorded).

(c) Remember to attach today's classroom worksheet!