Last name ________________________

First name ________________________

LARSON—MATH 353–CLASSROOM WORKSHEET 26

$x = n^2 + 1$ Primes Property Conjectures Investigation.

Set up.

1. Start the Chrome browser.
2. Go to `https://cocalc.com`
3. Log in to your account.
4. You should see an existing Project for our class. Click on that.
5. Make sure you are in your Home directory (if you put files in the Handouts directory they could be overwritten.)
6. Click "New", then "Jupyter Notebook", then call it **353-c26**.
7. Make sure you have SAGE as the *kernel*.
8. Look in your `Home` directory. You should see a `conjecturing.py` file and an `expressions` file **AND** today's Jupyter notebook.
9. Copy the latest version of `number_theory.sage` from the Handouts directory to your Home directory.
10. `load("number_theory.sage")`.

Review

1. The **research question** is: are the infinitely many primes of the form $x = n^2 + 1$?
2. What is a *property* in mathematics?
3. What is a *necessary condition* in mathematics?
4. What is a *sufficient condition* in mathematics?

Conjecture Proof Idea

```
count_prime_divisors(x) <= digits10(x)
```

We interpreted this as: for any $x = n^2 + 1$ prime, the number of distinct prime factors of x is no more than the number of base-10 digits. Here's a new idea to try: consider numbers $x < 10^1$, $x < 10^2$,..., $x < 10^k$. Maybe we can use this idea to make an inductive hypothesis?

Idea: Can we develop a theory for which $x = n^2 + 1$ integers are prime (that is, have the *property* of being prime?) Maybe generating necessary (and sufficient) condition conjectures for being prime will advance this idea?

1. The currently coded (and loaded) properties are:

```
properties = [is_prime, is_even, is_odd, is_abundant, is_deficient,
is_perfect, is_abundant_base, is_deficient_base, is_perfect_base,
is_semiprime, is_semiprime_base,
count_divisors_base_less_largest_prime_base,
radical_base_less_euler_phi_base,
count_divisors_base_less_smallest_prime_base,
smallest_prime_less_euler_phi_base,
smallest_prime_less_count_prime_divisors,
smallest_prime_less_count_prime_divisors_base,
euler_phi_base_less_sigma_base,
count_divisors_base_less_euler_phi_base,
largest_prime_base_less_divisors_base,
largest_prime_base_less_euler_phi_base, radical_base_less_divisors_bas
radical_base_less_sigma_base]
```

2. Try this sufficient condition run. Are the conjectures true? Can you find any counterexamples?

```python
objects = [5, 17, 65, 901, 325, 170, 2210, 101, 4625, 197, 1025, 4357,
    2, 10610]

properties = [is_prime, is_even, is_odd, is_abundant, is_deficient,
is_perfect, is_abundant_base, is_deficient_base, is_perfect_base,
is_semiprime, is_semiprime_base,
    count_divisors_base_less_largest_prime_base,
    radical_base_less_euler_phi_base,
    count_divisors_base_less_smallest_prime_base,
    smallest_prime_less_euler_phi_base,
    smallest_prime_less_count_prime_divisors,
    smallest_prime_less_count_prime_divisors_base,
    euler_phi_base_less_sigma_base,
    count_divisors_base_less_euler_phi_base,
    largest_prime_base_less_divisors_base,
    largest_prime_base_less_euler_phi_base,
    radical_base_less_divisors_base, radical_base_less_sigma_base]

prop_of_interest = properties.index(is_prime)

theorems = []

conjs = propertyBasedConjecture(objects, properties, prop_of_interest,
    theory = theorems, sufficient = True, debug=True, time=20)

for conj in conjs:
    print(conj)

```

3. One idea to get new conjectures: eliminate all the even numbers greater than 2 from the objects list (they can't be prime). We only really need to investigate the odd $x = n^2 + 1$ integers. Which of these are prime?

4. What other integer properties can we find (from the internet, papers, books, ChatGPT, etc (that we might add to get better conjectures)?

Getting your classwork recorded

When you are done, before you leave class...

1. Click the "Print" menu choice (under "File") and make a pdf of this worksheet (html is OK too).
2. Send me an email (`clarson@vcu.edu`) with an informative header like "Math 353 - c26 worksheet attached" (so that it will be properly recorded).
3. Remember to attach today's classroom worksheet!