Last name ______________________

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LARSON—MATH 353–CLASSROOM WORKSHEET 23
Multiplicative Functions & Primitive Roots.

Review

1. What is a *multiplicative function*?
2. We proved that, if an integer p is prime, then $\mathbb{Z}/p\mathbb{Z}$ is a field. If an integer $n > 1$ is not prime, can $\mathbb{Z}/n\mathbb{Z}$ be a field?
3. What is a *primitive root* in $\mathbb{Z}/n\mathbb{Z}$ (for integer $n > 1$)?
4. What are the polynomials $k[x]$ over a field k?

Question: Is Euler's ϕ function multiplicative?

1. What have we shown so far?
2. Argue: if r and s are relatively prime positive integers (so $\gcd(r, s) = 1$) then $\phi(rs) = \phi(r)\phi(s)$.
3. Argue: if an integer p is prime then $\phi(p^n) = p^n(1 - \frac{1}{p})$.
4. Argue: if an integer $n = p_1^{n_1} \ldots p_k^{n_k}$ (for primes $p_1 < \ldots p_n$) then

$$\phi(n) = n \prod_{p|n} (1 - \frac{1}{p}).$$

5. What are the polynomials $k[x]$ where k is the field $\mathbb{Z}/3\mathbb{Z}$?

6. (**Prop. 2.5.3), Root Bound**). If $f \in k[x]$ is a non-zero polynomial over a field k with degree $deg(f)$ then f has at most $deg(f)$ roots (elements α of the field k where $f(\alpha) = 0$).

7. Check that $f = x^2 - 1$ has exactly 2 roots where $f \in (\mathbb{Z}/3\mathbb{Z})[x]$.

8. Check that $f = x^3 - 1$ has exactly 3 roots where $f \in (\mathbb{Z}/7\mathbb{Z})[x]$.

9. (**Prop. 2.5.5)**) If p is prime and $d|(p-1)$ then $f = x^d - 1$ has exactly d roots.