Last name	
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LARSON—MATH 353-CLASSROOM WORKSHEET 23 Multiplicative Functions & Primitive Roots.

Review

- 1. What is a multiplicative function?
- 2. We proved that, if an integer p is prime, then $\mathbb{Z}/p\mathbb{Z}$ is a field. If an integer n > 1 is not prime, can $\mathbb{Z}/n\mathbb{Z}$ be a field?
- 3. What is a primitive root in $\mathbb{Z}/n\mathbb{Z}$ (for integer n > 1)?
- 4. What are the polynomials k[x] over a field k?

Question: Is Euler's ϕ function multiplicative?

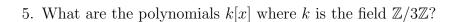
1. What have we shown so far?

2. Argue: if r and s are relatively prime positive integers (so gcd(r, s) = 1) then $\phi(rs) = \phi(r)\phi(s)$.

3. Argue: if an integer p is prime then $\phi(p^n) = p^n(1 - \frac{1}{p})$.

4. Argue: if an integer $n = p_1^{n_1} \dots p_k^{n_k}$ (for primes $p_1 < \dots p_n$) then

$$\phi(n) = n \prod_{p|n} (1 - \frac{1}{p}).$$



6. (**Prop. 2.5.3**), **Root Bound**). If
$$f \in k[x]$$
 is a non-zero polynomial over a field k with degree $deg(f)$ then f has at most $deg(f)$ roots (elements α of the field k where $f(\alpha) = 0$).

7. Check that
$$f = x^2 - 1$$
 has exactly 2 roots where $f \in (\mathbb{Z}/3\mathbb{Z})[x]$.

8. Check that
$$f = x^3 - 1$$
 has exactly 3 roots where $f \in (\mathbb{Z}/7\mathbb{Z})[x]$.

9. (**Prop. 2.5.5**)) If p is prime and
$$d|(p-1)$$
 then $f=x^d-1$ has exactly d roots.