Last name _____

First name _____

LARSON—MATH 353–CLASSROOM WORKSHEET 13
$n^2 + 1$ Primes Investigation.

1. Sign in to your CoCalc account.

   (a) Start the Chrome browser.

   (b) Go to `https://cocalc.com`

   (c) Log in to your account.

   (d) You should see an existing Project for our class. Click on that.

   (e) Make sure you are in your Home directory (if you put files in the Handouts directory they could be overwritten.)

   (f) Click "New", then "Jupyter Notebook", then call it **353-c13**.

   (g) Make sure you have SAGE as the *kernel*.

   (h) Look in your `Home` directory. You should see a `conjectures.py` file and an `expressions` file **AND** today's Jupyter notebook.

   Review

2. **Property Conjectures** example.

   For integers (or any other object-type) we can conjectures necessary or sufficient conditions for an integer to have that property.

```
properties = [is_prime, is_even]
property_of_interest = properties.index(is_prime)
objects = [3]
propertyBasedConjecture(objects, properties, property_of_interest)
```

3. **Facts about the Produced Conjectures**

   (1) **Truth**. They are TRUE for every input object.
   (2) **Significance**. Each conjecture, when added to the list of conjectures, was "better" for at least one input object than any previously stored conjecture.

   Experiments

   The **question** is: are the infinitely many primes of the form $n^2 + 1$?

4. To use the `Conjecturing` program, we'll need some *invariants*. The ones we've coded in class are in the *number_theory.sage* file in your Handouts folder. We don't want to keep re-coding those. We can use this file as a permanent record of everything we've coded for this research. Copy or move this file to your Home directory.

5. We'll want some data. Let's make a list L with all the primes up to 1000. How can we do that? We don't want to use all these when we produce conjectures, but we can use them to check if a given conjecture is true.

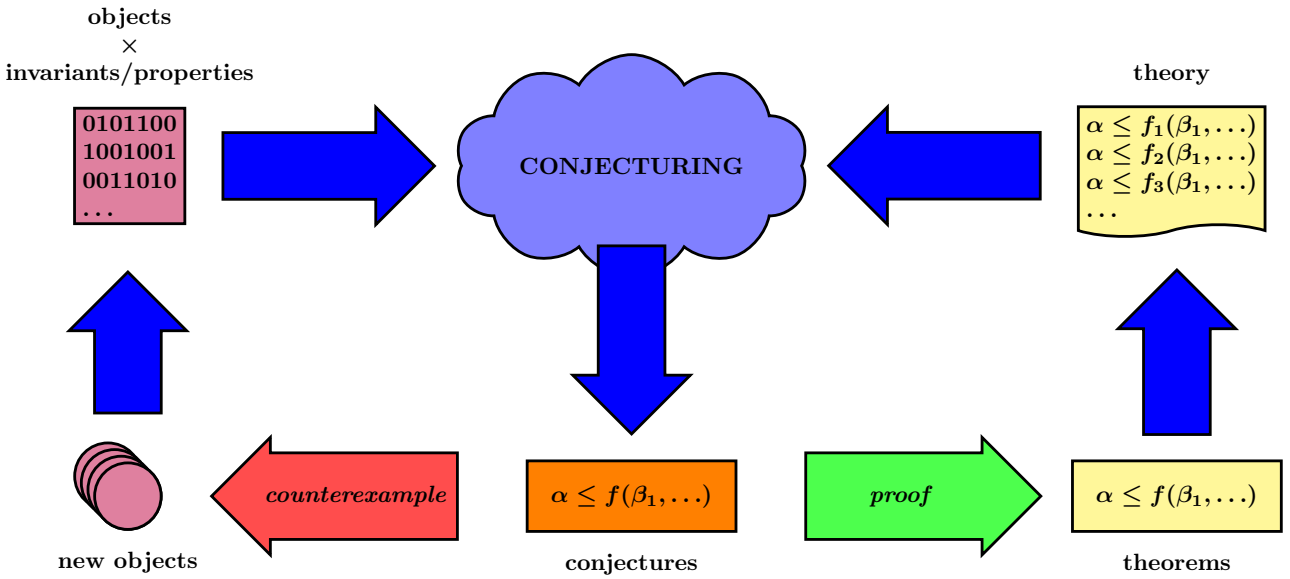
6. Let's use only a few of these as "objects" for conjecturing (fewer objects means less overfitting and shorter, more readable, conjectures).

7. What invariant should we make conjectures for? Since we want *primes*, but not all $n^2 + 1$ integers are prime, maybe the "number of prime divisors" would be a good invariant to start with. A *theory* of the number of prime divisors of $n^2 + 1$ integers might tell us something about our motivating question.

8. What command should we write to run the `Conjecturing` program?

9. On each iteration either all the conjectures are true (can we prove one?), or there is a false conjecture (can we find a counterexample)?

10. How should we proceed?



Getting your classwork recorded
When you are done, before you leave class...

1. Click the "Print" menu choice (under "File") and make a pdf of this worksheet (html is OK too).

2. Send me an email (`clarson@vcu.edu`) with an informative header like "Math 353 - c13 worksheet attached" (so that it will be properly recorded).

3. Remember to attach today's classroom worksheet!