Last name _____

First name _____

LARSON—MATH 353–CLASSROOM WORKSHEET 07
Number Theory in CoCalc/SAGE.

1. Sign in to your CoCalc account.

   (a) Start the Chrome browser.

   (b) Go to `https://cocalc.com`

   (c) "Create new account" using **your VCU email address** .

   (d) You should see an existing Project for our class. Click on that.

   (e) Make sure you are in your Home directory (if you put files in the Handouts directory they could be overwritten.)

   (f) Click "New", then "Jupyter Notebook", then call it **353-c07**.

   (g) Make sure you have SAGE as the *kernel*.

Review

   (a) What operator will give you the *remainder* of a divided by b?

   (b) What operator will give you the *quotient* of a divided by b?

   (c) What does `prime_pi` do?

   (d) Code and run: `plot(prime_pi, 1,1000, rgbcolor=(0,0,1))`.

   (e) Find the first few Euclidean primes. Let $P_1 = 2$. Then at each step find the product of the existing primes plus 1. Add the largest prime factor that is not in your current list of Euclidean primes.

   (f) Assuming unique factorization into primes (the Fundamental Theorem of Arithmetic), how does this construction give a proof that there are infinitely many primes?

Prime Number Theorem

2. (**Density of the Primes**). Find the ratio of the number of primes in the interval $[10^i]$ to 10^i for $i = 1 \ldots 9$.

3. What is the **Prime Number Theorem**?

4. Make a combined plot of the prime counting function and the ratio in the Prime Number Theorem.

The Ring of Integers Mod n

5. We can define the *ring of integers mod* 3 ($\mathbb{Z}/3\mathbb{Z}$) in Sage with `R = Integers(3)`. To see what they look like, try `list(R)`.

6. 1 in every ring is different. In $\mathbb{Z}/3\mathbb{Z}$, $1 + 1 + 1 = 0$. To see this in Sage, we need to tell it *which* "1" we mean. Try $a = R(1)$ and then evaluate $a + a + a$.

7. What is the *multiplicative order* of an element in $\mathbb{Z}/10\mathbb{Z}$?

8. Code and run:

```
R = Integers(10)
a = R(3)
a.multiplicative_order()
```

9. Now let's see what the powers of s look like. Evaluate: `[a**i for i in range(15)]`.

10. What is Euler's ϕ function?

11. Evaluate: `euler_phi(2007)`.

Getting your classwork recorded

When you are done, before you leave class...

(a) Click the "Print" menu choice (under "File") and make a pdf of this worksheet (html is OK too).

(b) Send me an email (`clarson@vcu.edu`) with an informative header like "Math 353 - c07 worksheet attached" (so that it will be properly recorded).

(c) Remember to attach today's classroom worksheet!