

Last name _____

First name _____

LARSON—MATH 350—HOMEWORK WORKSHEET 08
Test 1 Review

You should know the following definitions, theorems, algorithms, and proofs for the test. Write out careful definitions, theorem statements, algorithms, proofs, and solutions. Turn these in at test time.

Definitions, Notation & Tools. Write each definition and give an example.

1. What is a *permutation*?
2. What is a *combination*?
3. What is $\binom{n}{k}$ (how is this notation defined)?
4. What is the *Multiplication Principle*?
5. What is the *Pigeonhole Principle*?
6. What is *Mathematical Induction* (how does it work)?
7. What is the *Fibonacci sequence* (what is the definition)?

Theorems. State and give an example, application or confirmation.

8. What did we prove $\binom{n}{k}$ equals (what's the formula we found)?
9. What is the *Binomial Theorem*?

Problems. Explain your answers.

10. There are 6 guests at a birthday party. Each shakes the hand of every other (exactly once). How many handshakes occur?
11. Alice sits at a fixed spot of a round table. There are 5 other chairs around the table. How many ways are there to seat the 5 other guests?
12. A lottery ticket consists of a choice of 5 different numbers from 90 possible numbers. How many different choices are possible?
13. License plates consist of 3 letters, followed by 4 digits. How many license plates are possible?
14. In a hand of bridge each player is dealt 13 cards from a 52 card deck (where all 52 cards are different). How many different hands are possible?
Let $A = \{7, 8, 10, 11\}$, $B = \{a, b, c\}$.
15. Find $A \cup B$ and $|A \cup B|$.

16. Find $A \times B$.
17. True or False: $A \subseteq B$.
18. True or False: $|A \cup B| = |A| + |B| - |A \cap B|$.
19. True or False: $|A \cup B| = |A \setminus B| + |B \setminus A| + |A \cap B|$.
20. Find the decimal (base-10) representation for $(1010)_2$.
21. Find the binary (base-2) representation for 47.
22. List all the permutations of the letters a , b and c .
23. How many permutations are there of (the elements of) a 5-set?
24. How many 3-subsets are there of a 5-set?
25. List all 3-subsets of the set $\{A, B, C, D, E\}$.
26. Prove: $\binom{n}{0} + \binom{n}{1} + \dots + \binom{n}{n-1} + \binom{n}{n} = 2^n$ (for any integer $n \geq 1$).
27. Give a *combinatorial* (bijective) proof that $\binom{n}{k} = \binom{n}{n-k}$ (for $0 \leq k \leq n$).
28. Prove: $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$.
29. Determine the number of numbers in $[1000] = \{1 \dots 1000\}$ that are divisible by either 2 or 5.
30. 26 darts hit a 5 in. by 5 in. dart board. Argue that some pair of darts be within 1.5 in. of each other?
31. Use the Binomial Theorem to write out $(x + 1)^5$.
32. The Binomial Theorem gives the expansion of $(x + y)^n$ as a sum of terms of the form $cx^i y^j$ (c is a coefficient). What is the term $cx^7 y^j$ in the sum for $(x + y)^{100}$?
33. Expand $(1 + (-1))^n$ using the Binomial Theorem—do not evaluate the binomial coefficients, but otherwise simplify. Rearrange so that only positive terms are on each side of your equality. Give a combinatorial interpretation of this identity.
34. Draw 9 layers of Pascal's Triangle. Next to it, draw a second triangle with the values of the coefficients from Pascal's Triangle.
35. Why is Pascal's Triangle vertically symmetric?
36. Find F_8 (from the Fibonacci sequence).

New

37. How many bits does 10^{100} have if written in base-2?
38. A staircase has 10 steps. You take 1 or 2 steps at a time. How many ways are there to go up the steps?