Last name _____

First name _____

LARSON—MATH 350—CLASSROOM WORKSHEET 23
Number Theory and Combinatorics

Review

- What does it mean for integer a to *divide* integer b (that is, $a|b$)?
- What is a *prime* number?
- If a, b are integers and $b = aq + r$ (for integers q, r with $0 \leq r < a$), what are q and r called?
- (**Claim:**) Every positive integer can be written as the product of primes.
- (**Claim:**) Every positive integer can be written *uniquely* as the product of primes.
- (**Claim:**) $\sqrt{2}$ is irrational.
- (**Claim:**) There are infinitely many primes.
- (**Claim:**) For every positive integer k, there exist k consecutive composite integers.
- **Fermat's Little Theorem** For any prime p, and any positive integer a:

$$p|(a^p - a)$$

New

1. We are given n natural numbers: $a_1, a_2, ..., a_n$. Show that we can choose a (nonempty) subset of these numbers whose sum is divisible by n.

2. What does it mean for integers a and b to be *relatively prime*?

3. We are given n numbers from the set $\{1, 2, ..., 2n-1\}$. Prove that we can always find two numbers among these n numbers that are relatively prime to each other.

4. How many numbers are there up to 1200 that are relatively prime to 1200?