Last name \_\_\_\_\_

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## LARSON—MATH 350—CLASSROOM WORKSHEET 16 Fibonacci Numbers!

## Review

- How is the *Fibonacci sequence*  $F_n$   $(n \ge 0)$  defined?
- What is the Golden Ratio  $\phi$ ?

The terms of the Fibonacci sequence are given by the formula:

$$F_n = \frac{1}{\sqrt{5}} \left[ \left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n \right].$$

1. Use the formula to find  $F_0$ .

2. Use the formula to find  $F_1$ .

3. Use the formula to find  $F_2$ .

4. What happens to  $(\frac{1-\sqrt{5}}{2})^n$  as  $n \to \infty$ ?

5. So find an approximation for  $F_n$  (actually the limit as  $n \to \infty$ ).

6. Then find an approximation for  $\frac{F_{n+1}}{F_n}$ .

So  $F_n$  is a geometric series (well, almost, in the limit!).

7. Check that the geometric series  $G_n = c\phi^n$   $(n \ge 0)$  is "Fibonacci like" in the sense that  $G_n = G_{n-1} + G_{n-2}$ .

8. Let  $\bar{\phi} = \frac{1-\sqrt{5}}{2}$ . Check that the geometric series  $\bar{G}_n = c\bar{\phi}^n$   $(n \in \mathbb{Z}^{\geq 0}, c \in \mathbb{R}^{\geq 0})$  is "Fibonacci like" in the sense that  $\bar{G}_n = \overline{G_{n-1}} + \overline{G_{n-2}}$ .