

Last name _____

First name _____

LARSON—MATH 350—CLASSROOM WORKSHEET 15
Fibonacci Numbers!

Review

- What is *Pascal's Triangle*? Why does the i^{th} row sum to 2^i ? Why are the “outside” terms all equal to 1? Does each “inside” term equal the sum of the two terms above it?

Fibonacci Numbers

1. How is the *Fibonacci sequence* F_n ($n \geq 0$) defined?
2. Write out the terms F_0 through F_{10} .

Let G_n be the sum of the terms of the Fibonacci sequence up to F_n .

3. Write out the terms G_0 through G_{10} .
4. Do you see a pattern? If so, describe what you see in English.
5. Now try to formulate a mathematical conjecture.
6. Can you prove it? If you try induction, carefully specify the base case, the induction hypothesis, and exactly what you will try to prove.

The terms of this sequence are given by the formula:

$$F_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1 + \sqrt{5}}{2} \right)^n - \left(\frac{1 - \sqrt{5}}{2} \right)^n \right].$$

7. Use the formula to find F_0 .

8. Use the formula to find F_1 .

9. Use the formula to find F_2 .

10. What happens to $\left(\frac{1-\sqrt{5}}{2}\right)^n$ as $n \rightarrow \infty$?

11. So find an approximation for F_n (actually the limit as $n \rightarrow \infty$).

12. Then find an approximation for $\frac{F_{n+1}}{F_n}$.

So F_n is a geometric series (well, almost, in the limit!).