Last name ______________________

First name ______________________

LARSON—MATH 350—CLASSROOM WORKSHEET 09
Mathematical Induction

Review

- We gave a *combinatorial* proof that $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$ (for $0 \leq k < n$).

- How does proof by *mathematical induction* work?

1. Find $2 + 4$.

2. Find $2 + 4 + 6$.

3. Find $2 + 4 + 6 + 8$.

4. Find $2 + 4 + 6 + 8 + 10$.

5. Conjecture a formula for the sum of the first n even integers.

6. Let $P(n)$ be the statement that the formula is true for the first n even integers. $P(1)$ is clearly true. You showed $P(2)$, $P(3)$, $P(4)$ and $P(5)$ are true. Assume $P(k)$, that this property holds for the first k even integers. Write $P(k)$.

7. We now need to show that the truth of $P(k)$ implies the truth of $P(k+1)$. Write out $P(k+1)$, namely what it is we must show.

8. Assume $P(k)$ is true. Argue that it then follows that $P(k+1)$ is also true. Conclude that $P(n)$ is true for all integers $n \geq 0$.

Estimating $n!$ (Sec. 2.2)

9. Estimate 100!.

10. Use proof by induction to show that $2^{n-1} \leq n! \leq n^n$ (for $n \geq 4$).

11. What is *Stirling's formula*?

Inclusion-Exclusion (Sec. 2.3)

12. Determine the number of numbers in $[100] = \{1 \dots 100\}$ that are divisible by either 2 or 5.

13. Determine the number of numbers in $[100] = \{1 \dots 100\}$ that are divisible by either 2, 3 or 5.