Last name ______________________

First name ______________________

LARSON—MATH 350—CLASSROOM WORKSHEET 09
Mathematical Induction

Review

- What is a *combination*?
- (**Notation**) What is $\binom{n}{k}$?
- **Theorem** $\binom{n}{k} = \frac{n!}{k!(n-k)!}$?
- We gave a *combinatorial* proof that:

$$\binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n-1} + \binom{n}{n} = 2^n.$$

- We gave a *combinatorial* proof that $\binom{n}{k} = \binom{n}{n-k}$ (for $0 \le k \le n$).

We checked that $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$ (for $0 \le k < n$) for some small concrete numbers.

1. Let's revisit the case where $n = 5$ and $k = 3$. Substitute these numbers into the claimed statement, and recalculate the binomial coefficients.

   Consider any 5-set S. We might consider $S = \{a, b, c, d, e\}$. If we want to count the 3-subsets of S they will either contain a **or not**.

2. Suppose a 3-subset contains a. So the 3-subset contains 2 other elements. List the possibilities. How many are there?

3. Suppose a 3-subset **does not** contain a. So the 3-subset contains 3 elements from $\{b, c, d, e\}$. List the possibilities. How many are there?

4. Can you see a *reason* for *why* the statement is true in general (can you give an argument)?

Mathematical Induction (Chp. 2)

5. Find $1 + 3$.

6. Find $1 + 3 + 5$.

7. Find $1 + 3 + 5 + 7$.

8. Find $1 + 3 + 5 + 7 + 9$.

9. Conjecture a formula for the sum of the first n odd integers.

10. Let $P(i)$ be the statement that the formula is true for the first i odd integers. $P(1)$ is clearly true. You showed $P(2)$, $P(3)$, $P(4)$ and $P(5)$ are true. Assume $P(k)$, that this property holds for the first k even integers. Write $P(k)$.

11. We now need to show that the truth of $P(k)$ implies the truth of $P(k+1)$. Write out $P(k + 1)$, namely what it is we must show.

12. Assume $P(k)$ is true. Argue that it then follows that $P(k+1)$ is also true. Conclude that $P(i)$ is true for all integers $i \geq 0$.