Last name _____

First name _____

LARSON—MATH 310–HOMEWORK WORKSHEET 12
Gaussian Elimination.

General Instructions

1. Write up a **neat** assignment on a **new sheet** of paper. (Do not cram your answers between the lines).
2. **Number** your problems so that it is easy to see what work matches the assigned problems.
3. Remember to **give examples** (you do not understand a concept unless you can provide an example of it).

Gaussian elimination operations. Use these same three operations from solving systems of equations to produce a matrix *in echelon form* that represents a system of equations with the same solutions:

1. Switch any two rows.
2. Add a multiple of one row to another.
3. Scale any row (by multiplying by a non-zero constant).

Gaussian elimination algorithm.

1. Switch a not-yet-processed row with a left-most non-zero entry a to the top of the not-yet-processeded rows.
2. Use this row, and *pivot* term a, to get 0's below a.
3. Repeat on remaining unprocessed rows.

At the termination of this algorithm, the produced matrix is guaranteed to be (1) in echelon form, (2) with all 0's below the main diagonal, and (3) with all 0 rows at the bottom.

(over)

Problems

$$\begin{bmatrix} 0 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 3 & 2 \\ 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 6 & 7 \\ 0 & 0 & 0 & 9 & 8 \end{bmatrix}$$

1. Use Gaussian elimination to produce an equivalent matrix in echelon form. Explain why your final matrix is in echelon form.

$$\begin{array}{rrrcl} x_1 & +0x_2 & +2x_3 & = & 1 \\ 3x_1 & -x_2 & +x_3 & = & 2 \\ 5x_1 & -x_2 & +5x_3 & = & 3 \end{array}$$

2. (a) Write this system as a matrix equation $A\hat{x} = \hat{b}$.
   (b) Write the augmented matrix corresponding to this system.
   (c) Apply our Gaussian elimination algorithm to get an echelon-form matrix..
   (d) Write the system of equations corresponding to this echelon matrix.
   (e) Solve this simpler system (or argue that there can't be any solutions).
   (f) Write your solution as a vector $\hat{x}$.
   (g) Check that this $\hat{x}$ is indeed a solution to $A\hat{x} = \hat{b}$ (by calculating $A\hat{x}$ using matrix-vector multiplication.