LARSON—MATH 310—Homework 7
Test 1 REVIEW

Write up answers a complete and detailed test review. That's due at test time.

Definitions. Define each concept **and** give an example.

1. *linear combination* of vectors.
2. *square* matrix.
3. *symmetric* matrix.
4. *transpose* of a matrix A.
5. *inverse* A^{-1} of a matrix A.
6. $\mathbb{R}$, $\mathbb{R}^2$, $\mathbb{R}^3$, $\mathbb{R}^n$.
7. *vector space.*
8. *column space* $C(A)$ of a matrix A.
9. *row space* $C(A^T)$ of a matrix A.
10. *null space* $N(A)$ of a matrix A.

Problems. Explain your answers.

1. Write this system in the matrix form $A\vec{x} = \vec{b}$.

$$x + y + z = 7$$

$$x + y - z = 5$$

$$x - y + z = 3$$

2. Find all solutions of this system.
3. Let $\vec{v} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Find $\vec{v} \cdot \vec{w}$.
4. Find $||\vec{v}||$.
5. Find a unit vector $\vec{u}$ in the same direction as $\vec{v}$.
6. Find the angle between $\vec{v}$ and $\vec{w}$.
7. Find a vector $\vec{u}$ which is perpendicular to $\vec{v}$.
8. Write $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$ as a linear combination of $\vec{v}$ and $\vec{w}$.

9. Find $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

10. Let $\vec{u} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$. Check if $\vec{u}$, $\vec{v}$, and $\vec{w}$ are co-planar? Explain.

11. Find a 3×3 elimination matrix E which adds (one times) the second row to the third row of a matrix A.

12. Find a 3×3 elimination matrix E which adds -2 times the first row to the third row.

13. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$, and $B = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$. Find AB.

14. Find:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 1 \\ 5 & -1 & 5 \end{bmatrix}$$

15. Write the augmented matrix corresponding to the following system.

$$\begin{array}{cccccc} x_1 & +0x_2 & +2x_3 & = & 1 \\ 3x_1 & -x_2 & +x_3 & = & 2 \\ 5x_1 & -x_2 & +5x_3 & = & 3 \end{array}$$

16. Use Gaussian Elimination (row operations on matrices of coefficients) to solve the above system. Clearly indicate your row operations.

17. Let

$$E_{13} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Find the inverse of E_{13}.

18. Find a 3×3 matrix P where (left) multiplication of A by P reverses the first and third rows of A.

19. Find the inverse of P.

20. Let $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$. Find the inverse of D.

21. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}$. Find an LU-factorization of A.

22. Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 5 \end{bmatrix}$. Find an LU-factorization of A.

23. Let $A = \begin{bmatrix} 1 & 9 \\ 0 & 3 \end{bmatrix}$. Find A^T, A^{-1}, $(A^T)^{-1}$ (if they exist).

24. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. Find A^T, A^{-1}, $(A^T)^{-1}$ (if they exist).

25. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. Find A^T, A^{-1}, $(A^T)^{-1}$ (if they exist).

26. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 7 \end{bmatrix}$. Find A^T, A^{-1}, $(A^T)^{-1}$ (if they exist).

27. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. and $\vec{v} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$. Find $\vec{v}^T A^T$.

28. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. Find $A^T A$.

29. Find AA^T.

30. Explain why $A^T A$ is always symmetric.

31. Explain why every vector $\vec{v}$ in the null space of a matrix A is orthogonal to every row $\vec{\rho}$ of A.

32. Define the column space of $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix}$.

33. Define the row space of $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix}$.

34. Find a vector $\vec{v}$ in $\mathbb{R}^3$ that is *not* in the row space of A.

35. Find the null space of A.

Let

$$A = \begin{bmatrix} 1 & 1 & 2 & 4 \\ 1 & 2 & 2 & 5 \\ 1 & 3 & 2 & 6 \end{bmatrix}.$$

36. Find the row-reduced echelon form (RREF) for A.

37. What is the rank of A?