

Last name _____

First name _____

LARSON—MATH 310—HOMEWORK WORKSHEET 06

Test 1 Review.

General Instructions

1. **You should know the following definitions, and corresponding examples, for the test. Write out careful definitions and problem solutions. Turn these in at test time.**
2. Write up a **neat** assignment on a **new sheet** of paper. (Do not cram your answers between the lines).
3. **Number** your problems so that it is easy to see what work matches the assigned problems.
4. Remember to **give examples** (you do not understand a concept unless you can provide an example of it).

Definitions. Write each definition **and give an example**.

1. What is a *set*?
2. What is a *subset* of a set? (What does $S_1 \subseteq S_2$ mean?)
3. What is the *Cartesian product* of sets S_1 and S_2 ?
4. What is a *function*? (What does $f : D \rightarrow F$ mean?)
5. What is a *field*?
6. What are the *complex numbers*?
7. Why do any non-zero complex numbers $a + bj$ have a multiplicative inverse?
8. What is a *vector*?
9. What are examples of n -vectors?
10. What are examples of D -vectors?
11. What is the *conjugate* of a complex number $a + bj$?
12. What is the *absolute value* of a complex number $a + bj$?
13. Give an example of a D -vector which uses pairs for its domain.
14. What is the definition for adding n -vectors?
15. What is the definition for multiplication of an n -vector by a scalar?
16. What is the *dot product* of two n -vectors?

17. Why is the dot product of n -vectors commutative?
18. What is a *linear combination* of vectors $\hat{v}_1, \dots, \hat{v}_n$?
19. What is a *convex combination* of vectors?
20. What is the *span* of vectors $\hat{v}_1, \dots, \hat{v}_n$?
21. Let \mathcal{V} be a set of vectors. What is a *generating set* of vectors for \mathcal{V} ?
22. What is \mathbb{R}^n ?
23. What are the *standard* generators for \mathbb{R}^n ?
24. What is a *vector space*? Give a definition and an example.
25. What is a *subspace*? Give a definition and an example.
26. What is the *span* of vectors? Give a definition and an example.
27. What are *standard generators*? Give a definition and an example.
28. What is a *homogeneous linear system*? Give a definition and an example.

Problems

29. What is the geometric interpretation of 2-vector addition? Explain and give an example.
30. What is scalar-vector multiplication? Explain and give an example for 2-vectors.
31. For vectors $\hat{v} = [-1, 3]$ and $\hat{u} = [0, 4]$, find the vectors $3\hat{v}$, $2\hat{v}$, and $3\hat{v} - 2\hat{u}$. Draw $3\hat{v}$, $2\hat{v}$, and $3\hat{v} - 2\hat{u}$ as arrows on the same coordinate system.
32. Let a, b be real numbers. Consider the equation $z = ax + by$. Show that there are two 3-vectors \hat{v}_1, \hat{v}_2 such that the set of points $[x, y, z]$ satisfying the equation is exactly the set of linear combinations of \hat{v}_1 and \hat{v}_2 .
33. Consider the equation $z = 3x + 4y + 5$. Show that there are three 3-vectors $\hat{v}_0, \hat{v}_1, \hat{v}_2$ such that the set of points $[x, y, z]$ satisfying the equation is exactly $\{\hat{v}_0 + \alpha_1\hat{v}_1 + \alpha_2\hat{v}_2 : \alpha_1, \alpha_2 \in \mathbb{R}\}$.
34. Show that \mathbb{R}^2 is a vector space.
35. Use *backward substitution* to solve the following linear system. Explain.

$$\begin{cases} x + y + z = 6, \\ 2y - 3z = -19, \\ 4z = 20. \end{cases}$$

36. Do the vectors $[1, 0], [0, 1]$ span \mathbb{R}^2 ? Explain.
37. Do the vectors $[-1, 1], [1, -1]$ span \mathbb{R}^2 ? Explain.
38. Do the vectors $[1, 1], [1, -1], [0, 1]$ span \mathbb{R}^2 ? Explain.
39. Write the vector $[1, 0]$ as a linear combination of $[1, 1], [1, -1], [0, 1]$.