Last name ______________________

First name ______________________

LARSON—MATH 310—CLASSROOM WORKSHEET 25
Eigenvalues and Eigenvectors for Symmetric Matrices.

Review

- If there is a non-zero vector $\vec{x}$ and scalar λ with $A\vec{x} = \lambda\vec{x}$ then λ is an **eigenvalue** of A and $\vec{x}$ is a corresponding **eigenvector**.
- If $A\vec{x} = \lambda\vec{x}$, then $A\vec{x} - \lambda\vec{x} = 0$, and $(A - \lambda I)\vec{x} = 0$. Since $\vec{x}$ is non-zero that means that $(A - \lambda I)$ is not invertible, that the RREF has a 0-row, and that $\det(A) = 0$.
- (**Claim**:) The eigenvalues of any symmetric matrix are real.
- (**Claim**:) Any symmetric matrix A can be written as $A = Q\Lambda Q^T$, where Λ is diagonal and Q is orthogonal. (This is the **Real Spectral Theorem**).

1. (**Claim**:) If A is a symmetric matrix then eigenvectors corresponding to different eigenvalues are orthogonal.

2. (**Claim**:) For any matrix A, the eigenvalues of A^TA and AA^T are non-negative.

Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.

A^TA will be significant when we investigate the *singular value decomposition* (SVD). We proved that A^TA is symmetric and has real, non-negative, eigenvalues.

3. Find A^TA.

4. Find $(A^TA - \lambda I)$.

5. Find $\det(A^TA - \lambda I)$.
   (λ is a variable—so your answer will have λs in it).

6. Solve $\det(A^TA - \lambda I) = 0$.

7. For each solution λ, write the equation $(A^TA - \lambda I)\vec{x} = 0$, and solve for $\vec{x}$.

8. Check that your eigenvalue-eigenvector pairs work!