Last name ______________________

First name ______________________

LARSON—MATH 310—CLASSROOM WORKSHEET 24
Eigenvalues and Eigenvectors for Symmetric Matrices.

Review

- If there is a non-zero vector $\vec{x}$ and scalar λ with $A\vec{x} = \lambda\vec{x}$ then λ is an **eigenvalue** of A and $\vec{x}$ is a corresponding **eigenvector**.
- If $A\vec{x} = \lambda\vec{x}$, then $A\vec{x} - \lambda\vec{x} = 0$, and $(A - \lambda I)\vec{x} = 0$. Since $\vec{x}$ is non-zero that means that $(A - \lambda I)$ is not invertible, that the RREF has a 0-row, and that $\det(A) = 0$.

Let $A = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}$.

1. We found that the eigenvalues of A were *real*. Do you remember how to find them?

2. How can you check that the eigenvalues of *any* symmetric 2×2 matrix are real?

3. (**Claim**:) The eigenvalues of any symmetric matrix are real.

4. We found eigenvectors corresponding to each eigenvalue of A and it turned out that they were orthogonal. Do you remember how to find them?

5. Normalize these vectors (so they have unit length) and form a matrix $Q = [\vec{x_1}\vec{x_2}]$ where the eigenvalue-eigenvector pairs are $\lambda_1, \vec{x_1}$ and $\lambda_2, \vec{x_2}$.

6. Check that Q is orthogonal.

7. Let Λ be the matrix with λ_1, λ_2 on the diagonal (and 0 for all other entries). Check that $A = Q\Lambda Q^T$.

8. (**Claim**:) Any symmetric matrix A can be written as $A = Q\Lambda Q^T$, where Λ is diagonal and Q is orthogonal. (This is the **Spectral Theorem**).

9. Let $A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$. Find the eigenvalues of A, corresponding eigenvectors, normalize them, write Q and Λ and check that $A = Q\Lambda Q^T$.