Last name ______________________

First name ______________________

LARSON—MATH 310—CLASSROOM WORKSHEET 23
Eigenvalues and Eigenvectors.

Review

- If there is a non-zero vector $\vec{x}$ and scalar λ with $A\vec{x} = \lambda\vec{x}$ then λ is an **eigenvalue** of A and $\vec{x}$ is a corresponding **eigenvector**.

Let $A = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}$.

If $A\vec{x} = \lambda\vec{x}$, then $A\vec{x} - \lambda\vec{x} = 0$, and $(A - \lambda I)\vec{x} = 0$.

Since $\vec{x}$ is non-zero that means that $(A - \lambda I)$ is not invertible, that the RREF has a 0-row, and that $\det(A) = 0$.

1. Find $(A - \lambda I)$.

2. Use the 2×2 determinant formula to find $\det(A - \lambda I)$.
   (λ is a variable—so your answer will have λs in it).

3. Solve $\det(A - \lambda I) = 0$.

4. For each solution λ, write the equation $(A - \lambda I)\vec{x} = 0$, and solve for $\vec{x}$.

5. Check that your eigenvalue-eigenvector pairs work!

Let $A = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$.

$A^T A$ will be significant when we investigate the *singular value decomposition* (SVD). We proved that $A^T A$ is symmetric. We will prove that symmetric matrices have real eigenvalues.

6. Find $A^T A$.

If $A^T A\vec{x} = \lambda\vec{x}$, then $A^T A\vec{x} - \lambda\vec{x} = 0$, and $(A^T A - \lambda I)\vec{x} = 0$. Since $\vec{x}$ is non-zero that means that $(A^T A - \lambda I)$ is not invertible, that the RREF has a 0-row, and that $\det(A^T A) = 0$.

7. Find $(A^T A - \lambda I)$.

8. Use the 2×2 determinant formula to find $\det(A^T A - \lambda I)$.
   (λ is a variable—so your answer will have λs in it).

9. Solve $\det(A^T A - \lambda I) = 0$.

10. For each solution λ, write the equation $(A^T A - \lambda I)\vec{x} = 0$, and solve for $\vec{x}$.

11. Check that your eigenvalue-eigenvector pairs work!