Last name ______________________

First name ______________________

LARSON—MATH 310—CLASSROOM WORKSHEET 19
Projections and Orthogonal Bases.

Review

- What is the Fundamental Theorem of Linear Algebra?

We will find an orthogonal basis for the vector space spanned by vectors $\vec{a_1} = (3, 4, 4)$ and $\vec{a_2} = (2, 2, 1)$.

Let $\vec{p}$ be the projection of $\vec{a_2}$ on vector $\vec{a_1}$. Then $\vec{a_2}' = \vec{a_2} - \vec{p}$ is orthogonal to a_1.

We found that, for any matrix A (with linearly independent columns), that the projection $\vec{p}$ of a vector $\vec{b}$ onto the column space of A is:

$$\vec{p} = A\vec{x} \text{ where } \vec{x} = (A^T A)^{-1} A^T \vec{b}.$$

$$\text{so } \vec{p} = A(A^T A)^{-1} A^T \vec{b}.$$

1. Let A be the matrix formed by the single column $\vec{a_1}$.

2. Find A^T.

3. Find $A^T A$.

4. Find $(A^T A)^{-1}$.

5. Find $(A^T A)^{-1} A^T$.

6. Find $(A^T A)^{-1} A^T \vec{a_2}$.

7. Find $A(A^T A)^{-1} A^T \vec{a_2}$.

8. Find $\vec{a_2}' = \vec{a_2} - A(A^T A)^{-1} A^T \vec{a_2}$.

9. Check that $\vec{a_2}'$ and $\vec{a_1}$ are orthogonal. So $\vec{a_1}, \vec{a_2}'$ are an orthogonal basis for the space spanned by the original vectors $\vec{a_1}, \vec{a_2}$.

Now we will find an *orthonormal* basis by *normalizing* these vectors.

10. Find $||\vec{a_1}||$ and $\frac{\vec{a_1}}{||\vec{a_1}||}$, and $||\vec{a_2}'||$ and $\frac{\vec{a_2}'}{||\vec{a_2}'||}$.

11. Write a matrix Q whose columns are $\frac{\vec{a_1}}{||\vec{a_1}||}$ and $\frac{\vec{a_2}'}{||\vec{a_2}'||}$. This is an *orthogonal matrix.* Check that $Q^T Q = I$.