Last name ______________________

First name ______________________

LARSON—MATH 310—CLASSROOM WORKSHEET 18
Projections.

Review

- What is the *dimension* of a vector space?

A is an $m \times n$ matrix (m rows, n columns), with rank r. The **Four Subspaces** are $C(A)$, $C(A^T)$, $N(A)$ and $N(A^T)$.

Fundamental Theorem of Linear Algebra, Part 1

- (Rank Theorem) The column space and row space both have dimension r.

- (Rank-Nullity Theorem) The nullspaces have dimension n-r and m-r.

Fundamental Theorem of Linear Algebra, Part 2

- The row space of A and the null space of A are orthogonal.

  (So the row space of A^T and the null space of A^T are orthogonal. So...)

- The column space of A and the null space of A^T are orthogonal.

Let vector $\vec{b} = (3, 4, 4)$. We will *project* it onto the line through vector $\vec{a} = (2, 2, 1)$. We will find a vector $\vec{p}$ with the following 2 properties:

- (1) $\vec{p} = x\vec{a}$ for some scalar x.

- (2) $\vec{a}$ is orthogonal to $\vec{b} - \vec{p}$.

1. Write property (2) as an equation.

2. Substitute (1) into your equation.

3. Simplify, and solve for x.

4. What is $\vec{p}$?

5. Check that the vector $\vec{p}$ you found satisfies property (2).

Now we will project $\vec{b} = (3, 4, 4)$ onto the subspace containing vectors $\vec{a_1} = (2, 2, 1)$ and $\vec{a_2} = (1, 0, 0)$.

We will find a vector $\vec{p}$ with the following 2 properties:

- (1) $\vec{p} = x_1\vec{a_1} + x_2\vec{a_2}$ for some scalars x_1, x_2.
- (2) Each vector $\vec{a_1}$ and $\vec{a_2}$ is orthogonal to $\vec{b} - \vec{p}$.

6. Let A be the matrix with columns $\vec{a_1}$ and $\vec{a_2}$, let $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, and check that $\vec{p} = A\vec{x}$.

7. Find A^T and check that $A^T(\vec{b} - \vec{p}) = \vec{0}$, says the same thing as condition (2).

So, $A^T\vec{b} = A^T\vec{p} = A^TA\vec{x}$.

8. Find A^TA.

9. Now find $(A^TA)^{-1}$.