Last name ______________________

First name ______________________

LARSON—MATH 310—CLASSROOM WORKSHEET 16
Basis of a Vector Space

For each of the row space, column space and null space of a matrix A we want to find a linearly independent set of vectors whose linear combinations is that space, Such a set of vectors is called a *basis* for the space.

1. What is the *span* of a collection of vectors in a vector space?

2. What is a *basis* of a vector space?

Basis of the Row Space of a Matrix A

- Find the RREF of A.
- The non-zero rows of A are a basis for the row space.

Strang thinks of these as column vectors (so just transpose them: they live in $C(A^T)$).

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

3. Find a basis for the row space of A.

4. Find the rank of A.

5. What is the *dimension* of a vector space?

6. What is the dimension of $C(A^T)$?

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Basis of the Column Space of a Matrix A

- Find the RREF of A^T.
- The non-zero rows of A^T (transposed back to columns) are a basis for the column space.

7. Find a basis for the column space of A.

8. What is the dimension of $C(A)$?

Basis of the Null Space of a Matrix A

- Find the RREF of A. (This represents a simplified system of equations equivalent to $A\vec{x} = \vec{0}$).
- Write the corresponding (simple) system of equations.
- Write the pivot variables in terms of the free variables.
- Write the solution vectors in terms of only the free variables.
- Rewrite these as a linear combination of vectors where the free variables are coefficients (so you will have one vector per free variable).
- The vectors in this linear combination (which have no variables) are a basis for $N(A)$.

9. Find a basis for $N(A)$.