Last name ________________________

First name ________________________

LARSON—MATH 310—CLASSROOM WORKSHEET 15
Basis of a Vector Space

For each of the row space, column space and null space of a matrix A we want to find a linearly independent set of vectors whose linear combinations is that space, Such a set of vectors is called a *basis* for the space.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 5 & 6 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

1. Find the RREF for A.

2. Find the rank of A.

3. Show that the pivot rows are linearly independent. Thus they are a basis for the row space $C(A^T)$ of A

4. The *dimension* of the row space is the number of vectors in a basis. What is the dimension of $C(A^T)$?

5. Use the RREF to find the null space $N(A)$. The null space is a linear combination of the special vectors. Show that the special vectors are linearly independent. What is the dimension of the null space?

You can find a basis for the column space of A by applying these ideas to A^T.

6. Find the RREF for A^T.

7. Find the rank of A^T.

8. Show that the pivot rows are linearly independent. Thus they are a basis for the row space of A^T—which is also the column space $C(A)$ of A.

9. What is the dimension of $C(A)$?

10. Check that the dimension of the column space $C(A)$ and the dimension of the null space $N(A)$ sum to the number of columns of A.