Last name ______________________

First name ______________________

LARSON—MATH 310—CLASSROOM WORKSHEET 14
Row echelon form

Row-reduced echelon form (RREF) of a matrix will be a tool to help us give the complete description of the solutions of a system of linear equations. We will use our row operations to get as many columns as possible with all zeros above and below a non-zero *pivot*. To standardize, we go from left column to right column.

Call the reduced matrix R. It will consist of pivot columns and "free" columns. The *rank* of A is the number of pivots of A.

Let
$$A = \begin{bmatrix} 1 & 1 & 2 & 4 \\ 1 & 2 & 2 & 5 \\ 1 & 3 & 2 & 6 \end{bmatrix}.$$

We used the above procedure to reduce A to the RREF
$$R = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

1. What is the rank of A?

2. Write the pivot columns.

3. Explain why the pivot columns are linearly independent.

4. Write the free columns.

5. Show that each free column is a linear combination of pivot columns.

Let

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 4 & 8 \\ 4 & 8 & 6 & 8 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} 4 \\ 2 \\ 10 \end{bmatrix}.$$

6. Form the augmented matrix corresponding to the system $A\vec{x} = \vec{b}$.

7. Get this matrix to row-reduced echelon form.

8. Identify the pivot variables and the free variables.

9. Write the pivot variables in terms of the free variables.

10. Set the free variables to 0 to get a *particular solution* $\vec{x_p}$ of $A\vec{x} = \vec{b}$.

11. **Check** that $\vec{x_p}$ is a solution to the original system.